These two exercises were extracted from Dr. Vaaler's Spring 2002 Fourier Analysis Course. Wheeden and Zygmund's

(1) Let $f : \mathbb{R} \longrightarrow \mathbb{C}$ be a period-1 function of bounded variation. Then

$$\left|\widehat{f}(n)\right| \leq \frac{V_f}{2\pi\,|n|}$$

for all $n \in \mathbb{Z} - \{0\}$, where V_f is the total variation of f on $\mathbb{R}/\mathbb{Z}$.

(2) Let $f : \mathbb{R}/\mathbb{Z} \to \mathbb{C}$ be absolutely continuous on each closed interval $[\alpha, \beta] \subset (0, 1)$ and assume that

$$\int_0^1 x\left|f'(x)\right|\,dx < \infty.$$

Then

$$\lim_{x \longrightarrow 0^+} x\,|f(x)| = 0.$$