Network Models from Petri Nets with Catalysts

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Petri Nets

\[ \tau_1 \quad \tau_2 \]
Markings

\[ 2A + 1B \]
Executions
Executions

\[ \tau_1 \quad \tau_2 \]

\[ \tau_1 \quad \tau_2 \]
Executions

\[ 2A + 1B \xrightarrow{\tau_1} 1A + 1C \]
Sequential Execution

\[ \tau_1 \rightarrow \tau_2 \rightarrow \tau_1 \rightarrow \tau_2 \]
Sequential Execution

\[ A + 1 \rightarrow B + 1 \rightarrow C + 2 \]
Sequential Execution

\[ 2A + 1B \xrightarrow{\tau_1} 1A + 1C \xrightarrow{\tau_2} 1A + 2B \]
Concurrent Execution
String Diagrams
String Diagrams
Catalysts

\[ \tau_1 \]

\[ \tau_2 \]
Catalysts

A

B

C

\[ \tau_1 \]

\[ \tau_2 \]

### Catalysts

A

B

C

\[ \tau_1 \]

A

B

B

C

C

A

\[ \tau_1 \]
Catalysts
FP is a coproduct

\[ FP = \bigsqcup_{c \in \text{Catalysts}} FP_c \]
Not monoidal subcategories
Premonoidal Structure on Subcategories

\[ A \otimes_1 A \]
Monoidal Grothendieck Construction

**Theorem (Vasilakopoulou, M.)**

If $\mathcal{X}$ is cocartesian monoidal, then the 2-category of categories which are fibre-wise monoidally opfibred over $\mathcal{X}$ is equivalent to the 2-category of categories which are globally monoidally opfibred over $\mathcal{X}$.

\[ f\text{OpFib}(\mathcal{X}) \cong g\text{OpFib}(\mathcal{X}) \]
Network Models

- $FP$ is monoidally opfibred over $\mathbb{N}[C]$
Network Models

- $FP$ is monoidally opfibred over $\mathbb{N}[C]$

- inverse monoidal Grothendieck construction to get an indexed category

\[
\mathbb{N}[C] \rightarrow \text{Cat}
\]
Network Models

- $FP$ is monoidally opfibred over $\mathbb{N}[C]$
- inverse monoidal Grothendieck construction to get an indexed category
  
  $\mathbb{N}[C] \rightarrow \text{Cat}$

- Let $S$ denote the free symmetric monoidal category functor
  
  $S[C] \rightarrow \mathbb{N}[C]$
Network Models

- $FP$ is monoidally opfibred over $\mathbb{N}[C]$

- inverse monoidal Grothendieck construction to get an indexed category

\[
\mathbb{N}[C] \rightarrow \text{Cat}
\]

- Let $S$ denote the free symmetric monoidal category functor

\[
S[C] \rightarrow \mathbb{N}[C]
\]

- composite is a monoidal indexed category

\[
S[C] \xrightarrow{i} \mathbb{N}[C] \xrightarrow{p} \text{Cat}
\]
Theorem (Baez, Foley, M.)

The global monoidal indexed category $G : S(C) \to \text{Cat}$ lifts to a functor $\hat{G} : S(C) \to \text{PreMonCat}$:

$$
\begin{array}{c}
\text{PreMonCat} \\
\downarrow U \\
\text{Cat}
\end{array}
\xrightarrow{\hat{G}}
\begin{array}{c}
S(C) \\
\downarrow G
\end{array}
$$
Network Models

- monoidal functor

\[
S[C] \xrightarrow{\hat{G}} \text{PreMonCat}
\]

- monoidal Grothendieck construction gives a monoidal category
  - objects = same objects as \( FP \), markings
  - morphisms = sequential executions + permutations of catalyst tokens
  - tensor = concurrent execution + permutation sum

This gives a variant of the category \( FP \) which models **individual token philosophy** on the catalyst tokens, and **collective token philosophy** on all others.
Future

- applications to queueing theory
Future

- applications to queueing theory
- Petri nets with guards
Future

- applications to queueing theory
- Petri nets with guards
- model individual token philosophy by mimicking the usual theory, but over a cocartesian base
John Baez, John Foley, and Joseph Moeller.  
Network models from Petri nets with catalysts.  

John Baez, John Foley, Joseph Moeller, and Blake Pollard.  
Network models.  

John Baez and Jade Master.  
Open Petri nets.  

Joe Moeller and Christina Vasilakopoulou.  
Monoidal Grothendieck construction.  