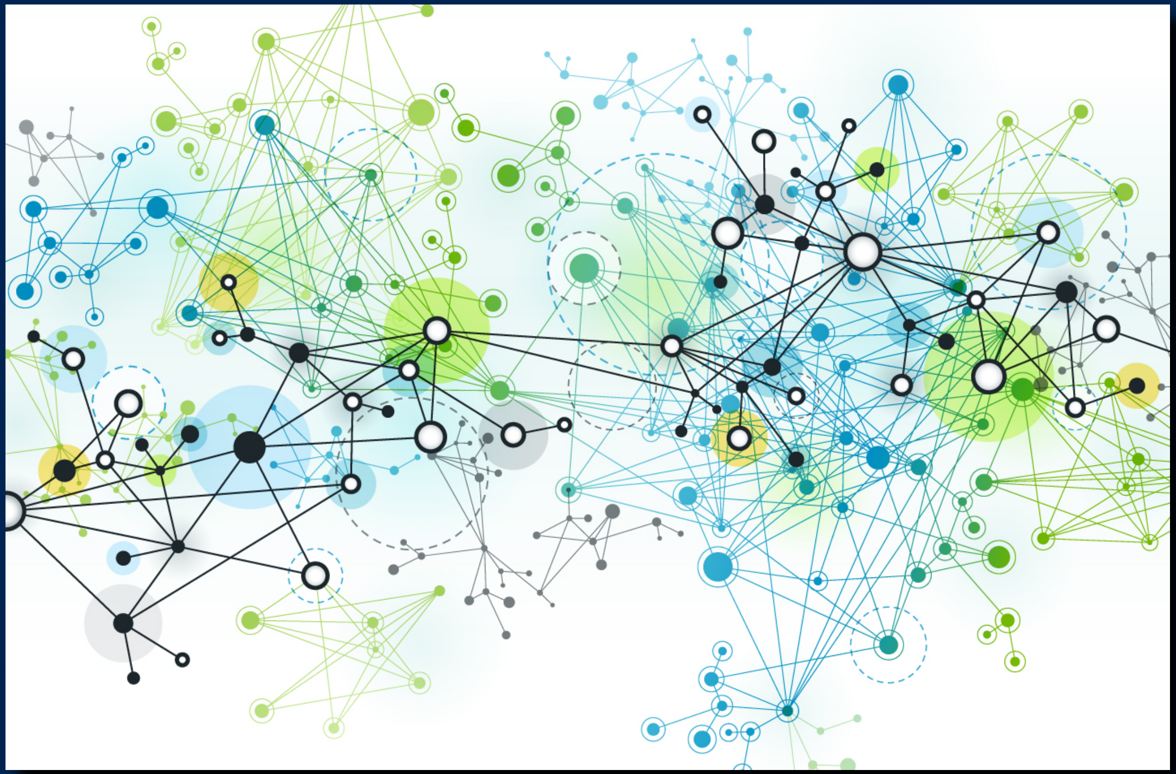
The Maximum Caliber Principle

for modeling stochastic dynamic processes



Goal: A statmech for dynamics

Input a model, predict force-flow distributions.

All Flows: Of molecules, energy, fluids, electrical currents. In trafficking networks in biochemistry, brains & ecology.

To derive "laws": Ohm's law (electrical), Fick's Law (particles), Fourier's Law (heat), Newtonian Fluids (momentum).

General applicability: Systems that are nonlinear; have large fluctuations; are Far From Equilibrium; are not thermal (like traffic).

First, *Equilibrium* statmech

MAXENT: Maximize the entropy, constrained by <E>

$$\epsilon_i \quad , \quad \text{model}$$

$$\langle E \rangle = \sum p_i \epsilon_i \tag{1}$$

$$S/k = -\sum p_i \ln p_i \tag{2}$$

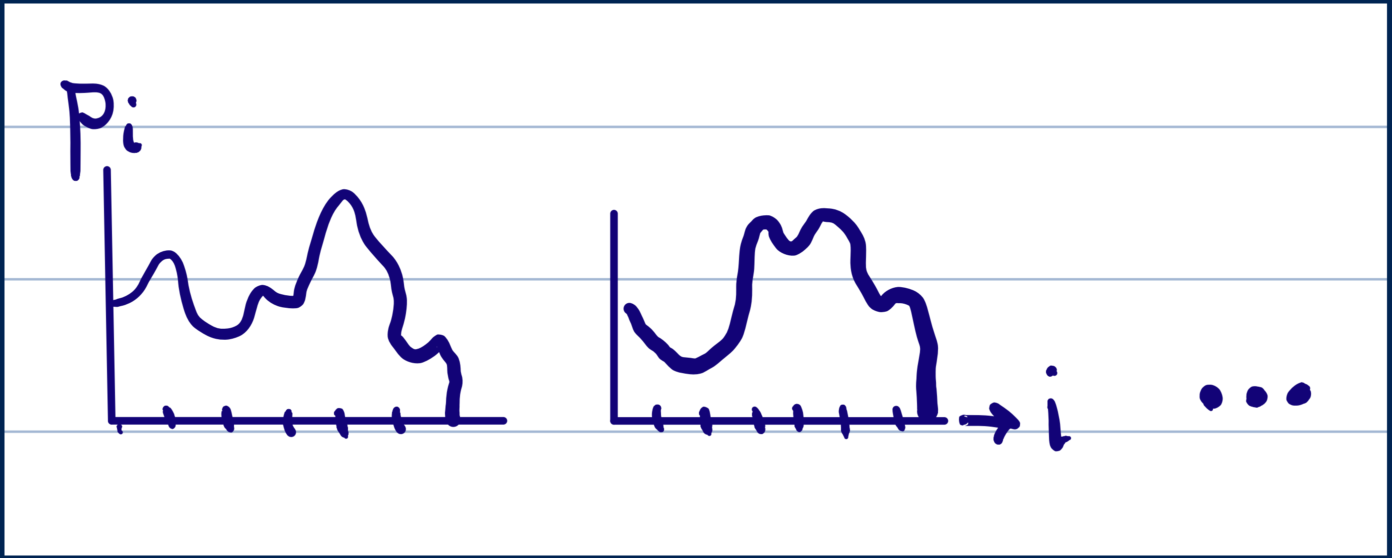
$$F = U - TS, \quad dF = 0 \tag{3}$$

$$q = \sum e^{-\epsilon_i/kT} \tag{4}$$

$$p_i = e^{-\epsilon_i/kT}/q \tag{5}$$

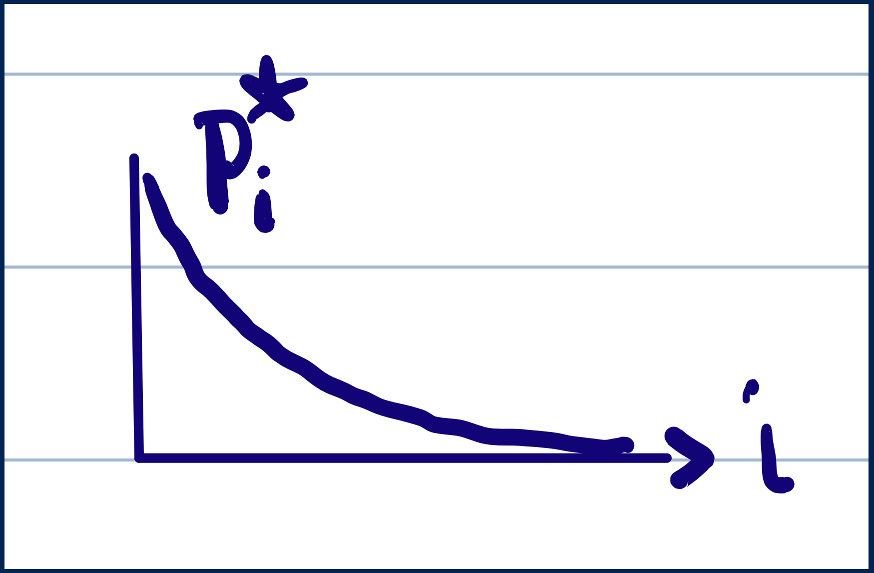
1) The *math* entropy

$$S_{\text{math}}\{p_i\} = -\sum_i p_i \ln p_i$$ ***For any*** $\{p_i\}$.



2) The ***MaxEnt*** *entropy*

$$S^*_{\mathrm{maxent}}\{p_i^*\} = -\sum_i p_i^* \ln p_i^* :$$ *For one* $\{p_i^*\}$.

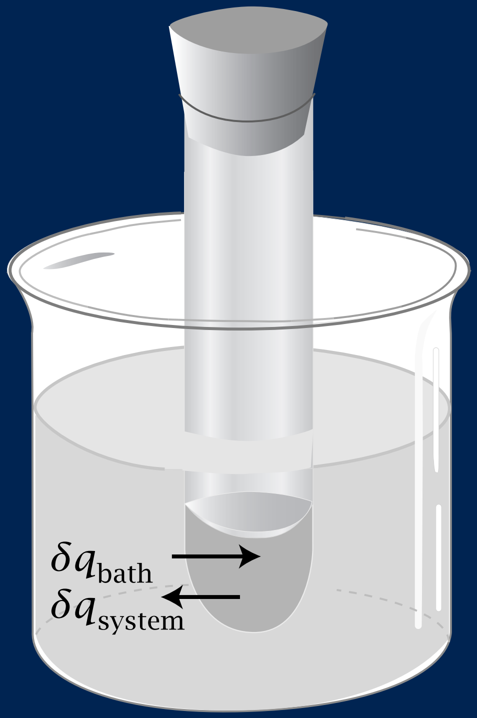


SHORE & JOHNSON 1980. Only this entropy is useful for prediction & inference.*

3) The *2nd Law* *entropy*

$$\Delta S_{\mathrm{2nd\ Law}} = \frac{\delta q_{\mathrm{rev}}}{T}$$

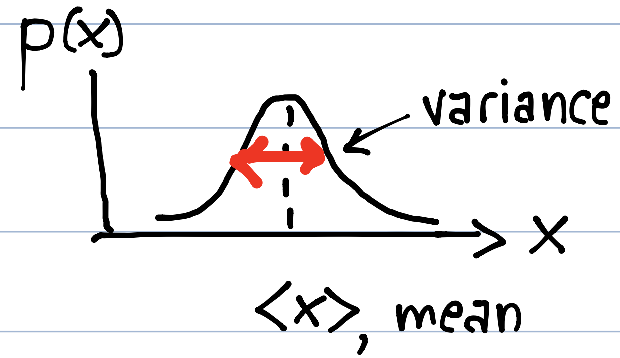
- Clausius 1854.
- It's measurable.
- Predicts equilibrium.
- Requires reversibility.
- It links energy to entropy.



The foundations of EQ statmech

Material properties come from microscopic distributions.

Beyond just averages. Sometimes, distributions matter.



- Kinetic theory of gases $\langle v^2 \rangle = 3kT/m$
- Heat capacities $C_V = \langle \Delta E^2 \rangle / kT^2$
- Random flights $\langle x^2 \rangle = 6Dt$

The brilliant insight of EQ Statmech:

$$\Delta S^*_{\text{maxent}}\{p_i\} = \Delta S_{2^{\text{nd}}\ \text{Law}} \quad !!!$$

<u>BUT:</u>

The LHS <u>requires</u> $\Delta S^*_{\text{maxent}}$.

And,   The RHS <u>requires</u> EQUILIBRIUM.

Problems with traditional NET

It is not a general method for model-making.

- Limited to near-equilibrium slow systems.
- Limited to linear force-flow processes.
- Doesn’t treat single- or few-agent systems.
- Doesn’t derive Phenomenological Laws.

Problem (1): *Entropy Production* has no meaning

- $S(t), \quad \dfrac{dS}{dt}, \quad S(t) = -\sum p(t) \ln p(t)\,.$
- dS/dt gives incorrect Kirchoff current law.

… except near equilibrium, where:

- $S(t) \rightarrow S^*$
- $p_i(t) \rightarrow p_i^* \rightarrow p_{\mathrm{Boltzmann}}$
- $S \rightarrow S(U, V, N) \Longrightarrow$ small gradients.
- No gradients $\Longrightarrow$ no Phenomenological Laws

Problem (2): Flows are linear functions of forces

$$\text{Dissipation} = \sum (F \times J) \approx J^2 .$$

- Ohm's Law: Power = RI^2
- Viscous fluids: Dissipation ~ $(\text{velocity})^2$

BUT THIS PRECLUDES:

Non-newtonian fluids, diodes, transistors,
feedback and delays,
Michaelis-Menten & Hill kinetics, …

Problem (3) Modeling noise

You can't just append random noise to a model.

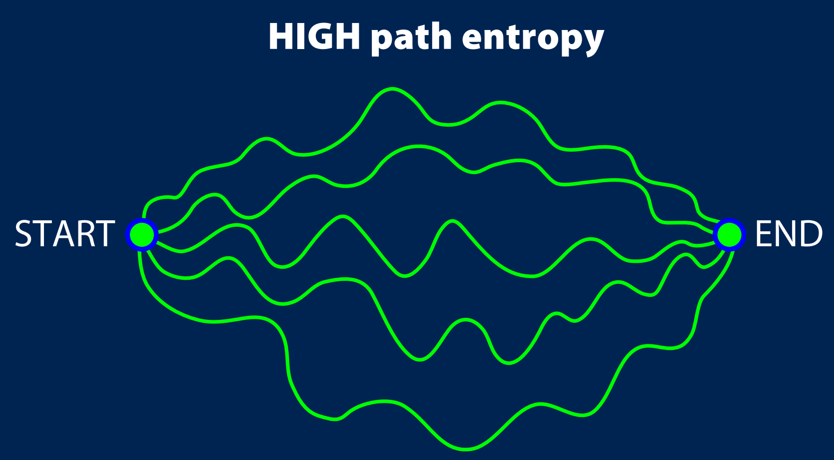
Langevin models don't work for nonlinear systems.

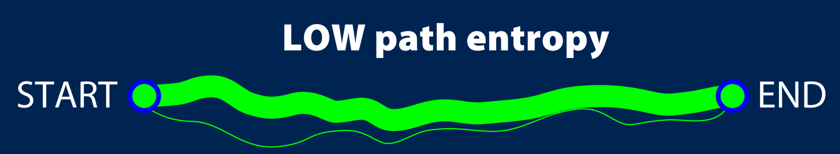
$$\left.\begin{aligned} m\frac{dv}{dt} + \xi v &= \\ \frac{dA}{dt} + f(A) &= \end{aligned}\right\} \text{Random noise term}$$

A Langevin diode will rectify its own fluctuations!
(van Kampen, 1981)

(4) MaxCal is about *path entropies*

not state entropies: $S^*_{\text{path}}\{p_i^*\} = -\sum_i p_i^* \ln p_i^* \,.$





Max Cal is *Max Ent for pathways*

Maximize path entropy, constrained by rates <J>.

j_i , model fluxes

$$\langle J \rangle = \sum p_i j_i \tag{1}$$

$$S = -\sum p_i \ln p_i \tag{2}$$

$$\mathcal{C} = S - \lambda \langle J \rangle, \quad d\mathcal{C} = 0 \tag{3}$$

$$q = \sum e^{-\lambda j_i} \tag{4}$$

$$p_i = e^{-\lambda j_i}/q \tag{5}$$

Is Max Cal the NESM Principle?

Some key confirmatory tests:

- Master equations (Stock et al, **JCP** `08).
- Green-Kubo (Hazoglou et al, **JCP** `15).
- Onsager recriprocal relations (``).
- Prigogine Minimum Entropy Production (``).
- Markov models (Ge et al, **JCP** `12).
- Kirchoff current law (Ghosh, **Ann Rev PChem** `20).

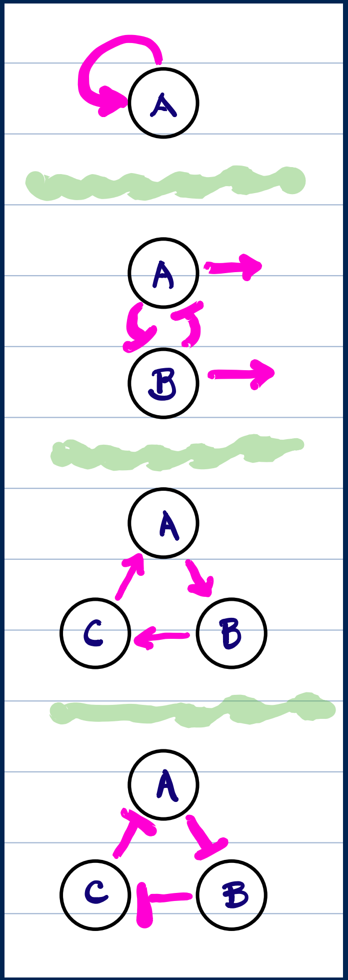
Some Max Cal modeling

- Fick's Law, FP (Ghosh, **AJP** `06, **JPCB** `07).
- Single-particle reactions (Wu **PRL** `09).
- 3-state molecular motors (Presse **PRE** `10).
- Find MD rxn coords (Tiwary & Berne **PNAS** `16).
- Dissipation in flows (Agozzino **PRE `19).**
- Inferring micro rates from avgs (Dixit **JCTC** `15).
- Infer networks from flows (Weistuch, **PLOSCB** `20).

Dynamical nonlinearities & fluctuations

Max Cal predicts the noise from the model.

- **Positive feedback** (Ghosh **BJ** `17, **JPCB `**18).
- **Toggle switch** (Presse **JPCB** `11, **Ann Rev PC** `20).
- **Molecular motors** (Presse **PRE** `10).
- **Repressilator** (oscillator, clock) (Ghosh **JPCB** `19).



Maximum Caliber:

A general inference principle

for dynamics and routes on pathways & networks.

Thanks to:

Rob Phillips
Luca Agozzino
Gerhard Stock
Steve Presse
Purushottam Dixit

Kings Ghosh
Corey Weistuch
Michael Hazoglou
Jason Wagoner
NSF

- S Presse …, *Rev Mod Phys,* 2013
- P Dixit …, *J Chem Phys,* 2018
- K Ghosh …, *Ann Rev Phys Chem,* 2020