Loop Quantum Gravity

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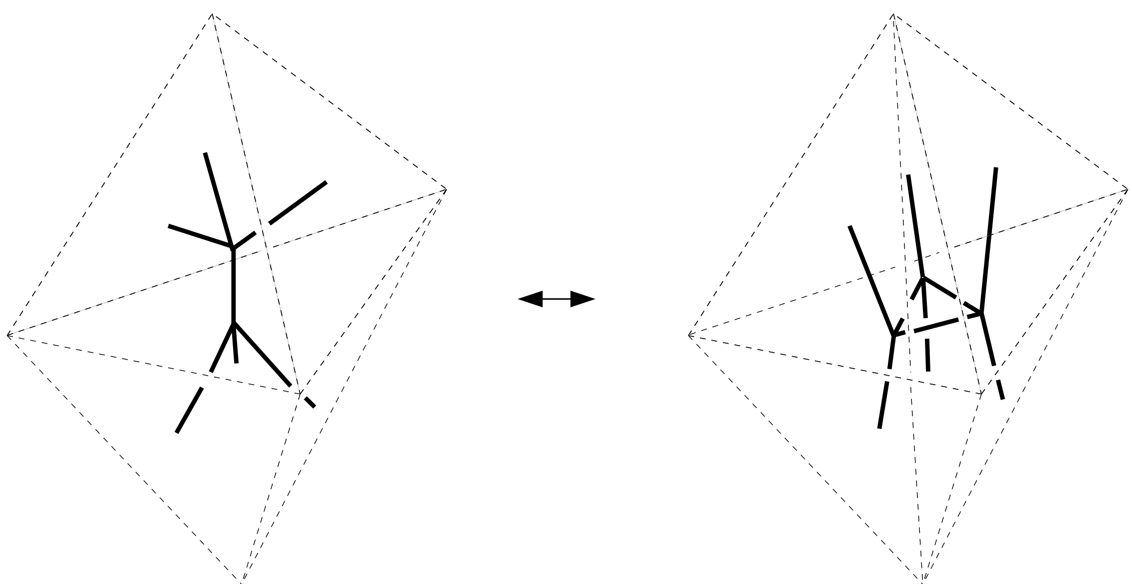


This talk and references can be found at:

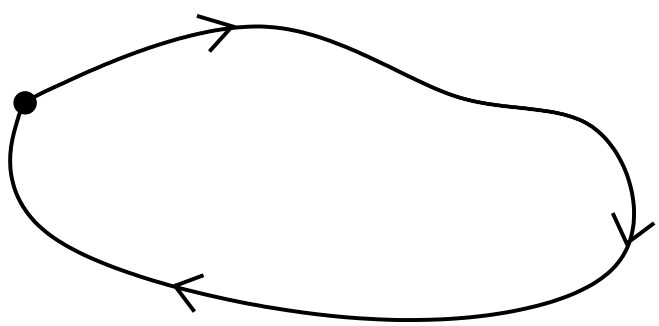
http://math.ucr.edu/home/baez/acm/

Loop Quantum Gravity

Loop quantum gravity tries to combine general relativity and quantum theory in a *background-free* theory. So, we cannot take gravitons, strings, etc. moving on a spacetime with a pre-established geometry as basic building blocks of the theory. Instead, we must start with *quantum states of geometry*.

To describe these, we ask:

What is the amplitude for a spinning test particle to come back to the state it started in when we parallel transport it around a loop in space?

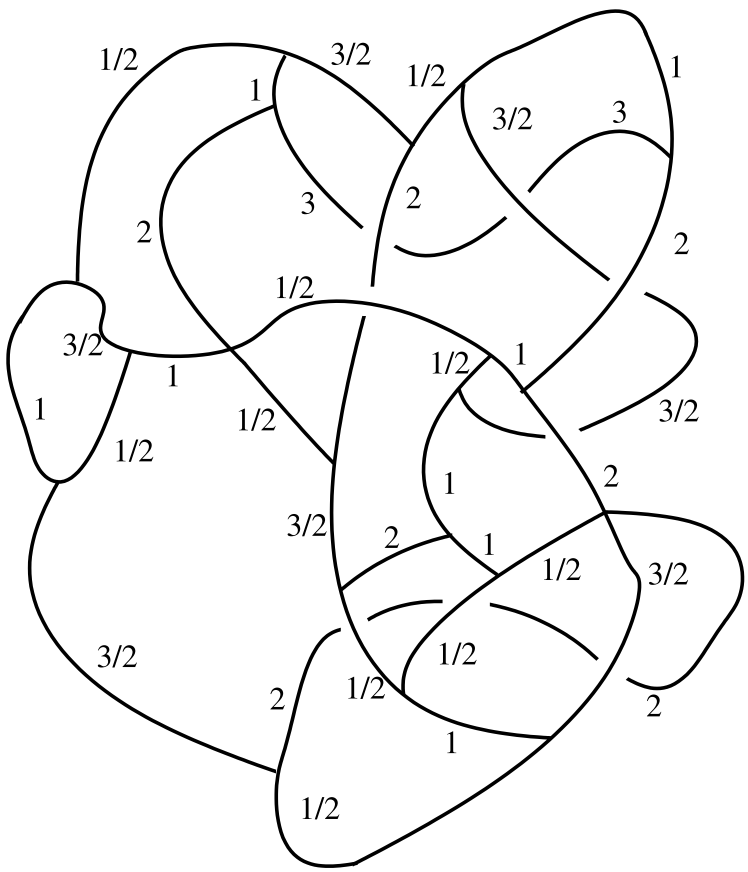


$j = 1/2, 1, 3/2, 2, \ldots$

The answer doesn't depend on the starting point or the direction of the loop, so we can ignore those. It's enough to consider spin-1/2 particles, so *a state of quantum geometry assigns to each loop an amplitude — a complex number.*

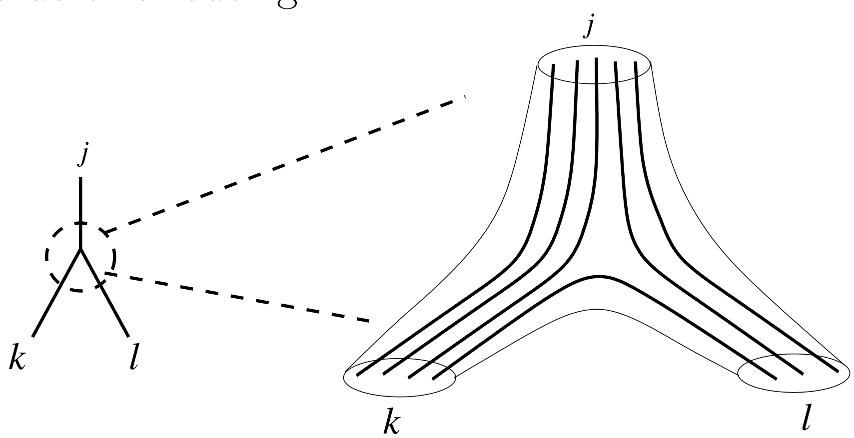
Spin Networks

More generally, a state of quantum geometry assigns an amplitude to any system of spinning test particles tracing out paths in space, merging and splitting. These are described by *spin networks*: graphs with edges labelled by spins...

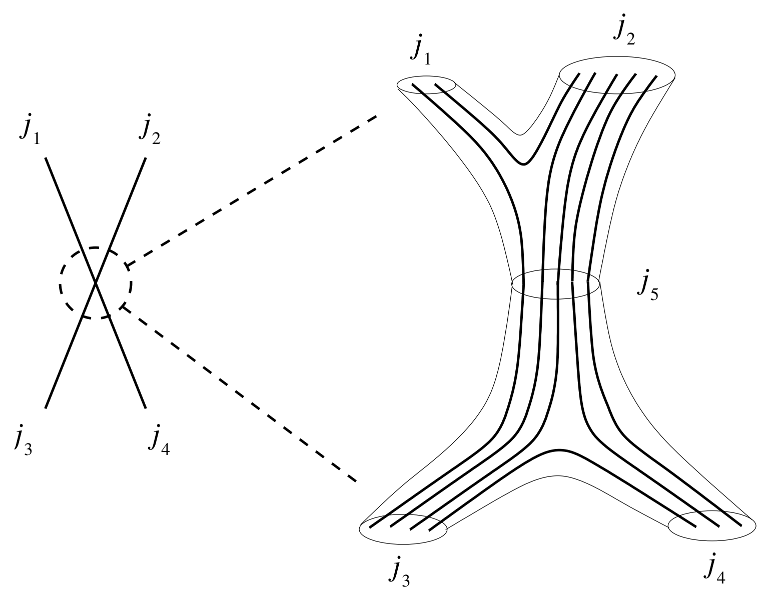


...together with 'intertwining operators' at vertices saying how the spins are routed. These are described using the mathematics of spin: the representation theory of the group SU(2). But we can also *draw* them!

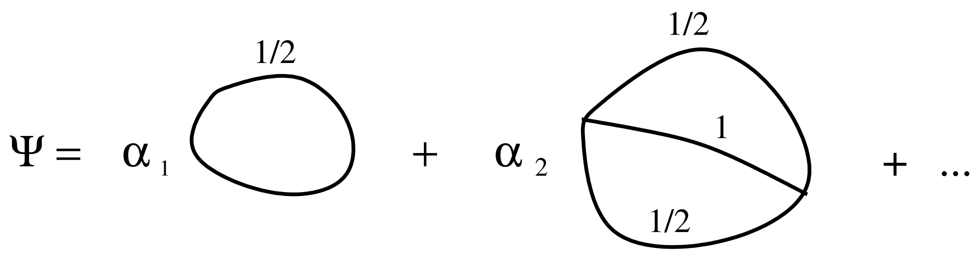
For vertices where 3 edges meet, there's at most one way to do this routing:



For vertices where more than 3 edges meet, we can formally 'split' them to reduce the problem to the previous case:



A quantum state of the geometry of space assigns an amplitude to any spin network. So, we can think of these states as *complex linear combinations of spin networks*, with these amplitudes as coefficients:



We could also use loops, but spin networks are an *orthonormal basis* of states, so they are more convenient.

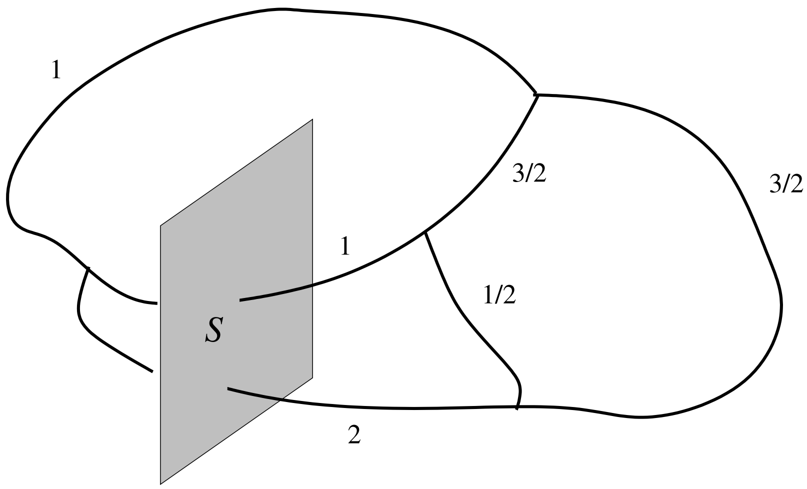
In this theory, the space around us is described by a huge linear combination of enormous spin networks — a complicated 'weave' that approximates the seemingly smooth geometry we see at distances much larger than the Planck length ($\sim 10^{-35}$ meters).

To see how this works, we need operators corresponding to interesting observables: lengths, areas, volumes...

Here we shall only consider *area operators*....

Quantization of Area

If a spin network intersects a surface S transversely:



then this surface has a definite area in this state, given as a sum over the spins j_e of the edges e poking through S:

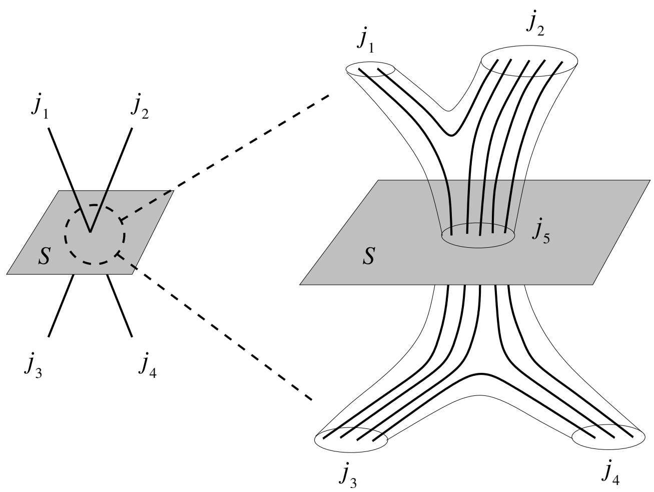
$$\text{Area}(S) = 8\pi\gamma \sum_{\text{edges } e} \sqrt{j_e(j_e+1)}$$

in units where the Planck length is 1. In particular, the operator for area has a *discrete spectrum!*

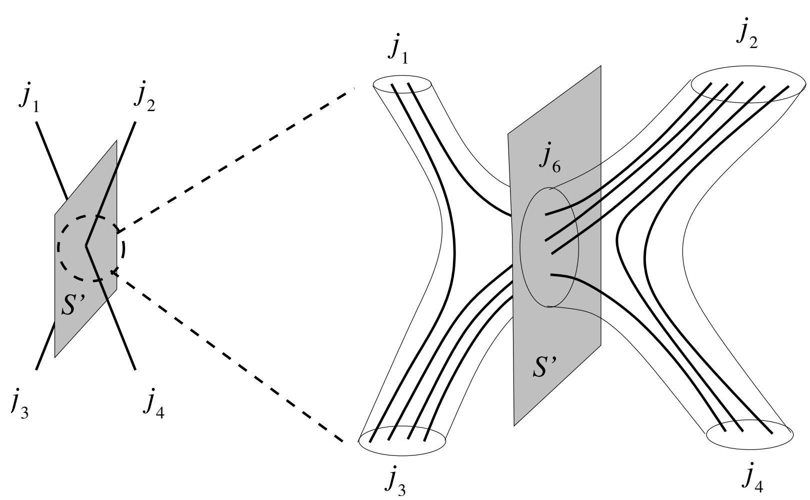
Here γ is a constant called the 'Barbero-Immirzi parameter'. So far we can only determine this by computing the entropy of a black hole in loop quantum gravity and comparing the answer to Hawking's calculation.

Uncertainty Principle for Area

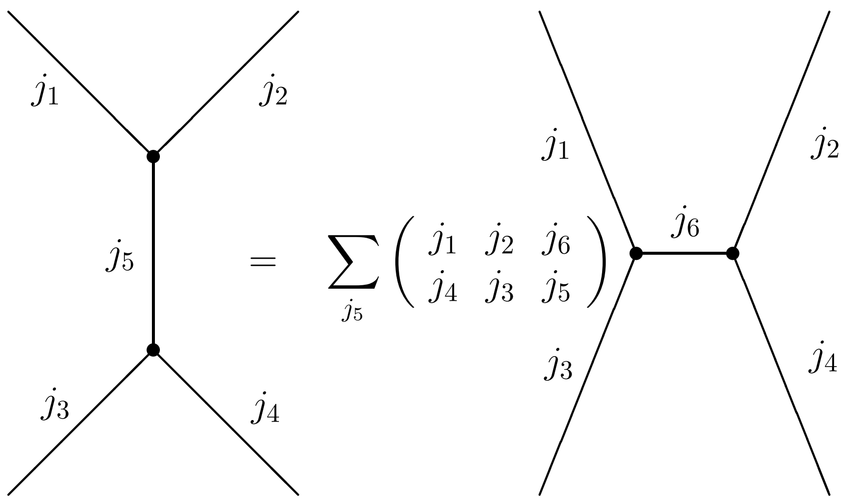
If a surface S intersects a spin network at a vertex, we must examine the routing to compute the area of S:



To describe states with definite areas, we must split the vertex so that the new edge intersects S transversely. This surface S' requires a different splitting:



Different splittings give different bases of states. To change from one basis to another we must use a matrix called the '$6j$ symbols':

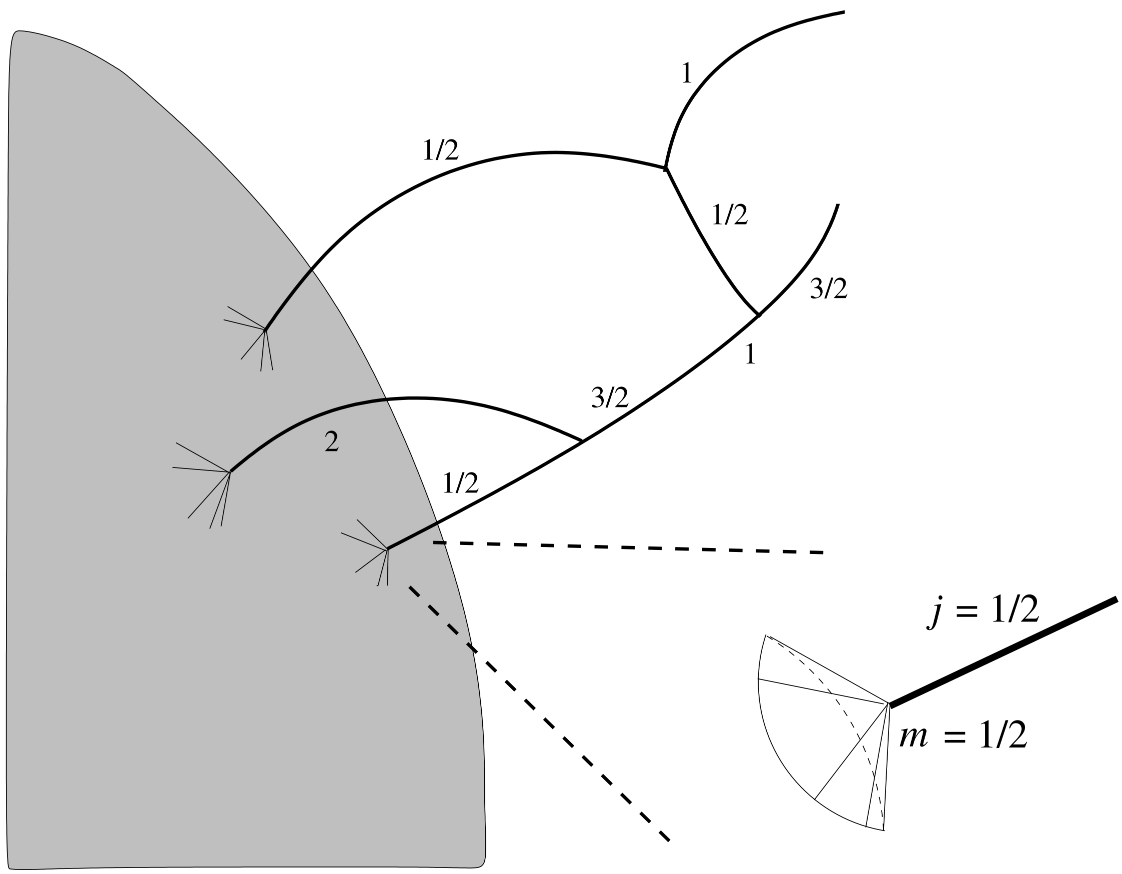


The area of S only has a definite value in the first basis of states, while that of S' only has a definite value in the second basis. *There is no basis of states in which the areas of both S and S' have definite values!*

In other words, the area operators for intersecting surfaces cannot be simultaneously diagonalized, so the uncertainty principle applies.

Black Hole Entropy

We can study loop quantum gravity in the presence of a uncharged, nonrotating black hole. Spin network edges puncturing the horizon contribute to its area. The intrinsic curvature of the horizon is concentrated at these punctures:



The angle deficit at a puncture is determined by a number

$$m = -j, -j+1, \ldots, j-1, j$$

where j is the spin of the edge piercing the horizon at this point.

A quantum state of the horizon is thus determined by two lists of numbers: j_i and m_i, with $j_i \in \{\frac{1}{2}, 1, \frac{3}{2}, \dots\}$ and $m_i \in \{-j_i, -j_i + 1, \dots, j_i\}$. If the black hole has area close to A, these lists must satisfy

$$\left| A - 8\pi\gamma \sum_i \sqrt{j_i(j_i+1)} \right| < \delta$$

for some number $\delta > 0$ — our error tolerance.

If we count the total number N of such states, for large A we find it grows about exponentially:

$$\ln N \sim (\gamma_0/\gamma)\frac{A}{4}$$

where γ_0 is independent of our error tolerance:

$$\gamma_0 = 0.27406685...$$

The black hole entropy is the logarithm of the number of states:

$$S = \ln N \sim (\gamma_0/\gamma)\,\frac{A}{4}$$

This matches Hawking's famous semiclassical calculation:

$$S = \frac{A}{4}$$

if and only if the Barbero–Immirzi parameter is given by

$$\gamma = \gamma_0.$$

Thus, *agreement with semiclassical results forces a specific value for the 'quantum of area'*: with $\gamma = \gamma_0$, the smallest allowed area is

$$8\pi\gamma\sqrt{\tfrac{1}{2}(\tfrac{1}{2}+1)} = 5.965222...$$

times the Planck length squared: about $1.5 \cdot 10^{-69}$ meters^2.

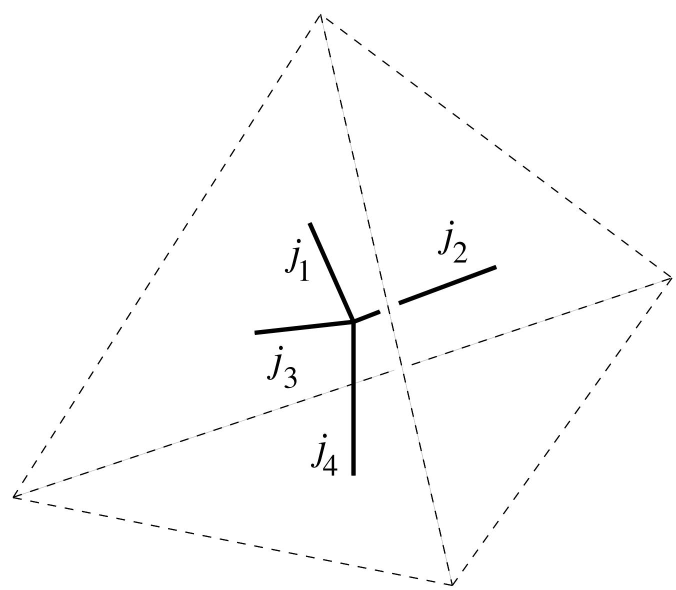
The same sort of calculation works for charged and/or rotating black holes, as well as black holes distorted by an external gravitational field — always with the same value of γ!

However, all this work is very tentative. Changing certain assumptions, we obtain different results. And we are not yet able to determine γ using *just* loop quantum gravity: we still need help from Hawking.

Dynamics

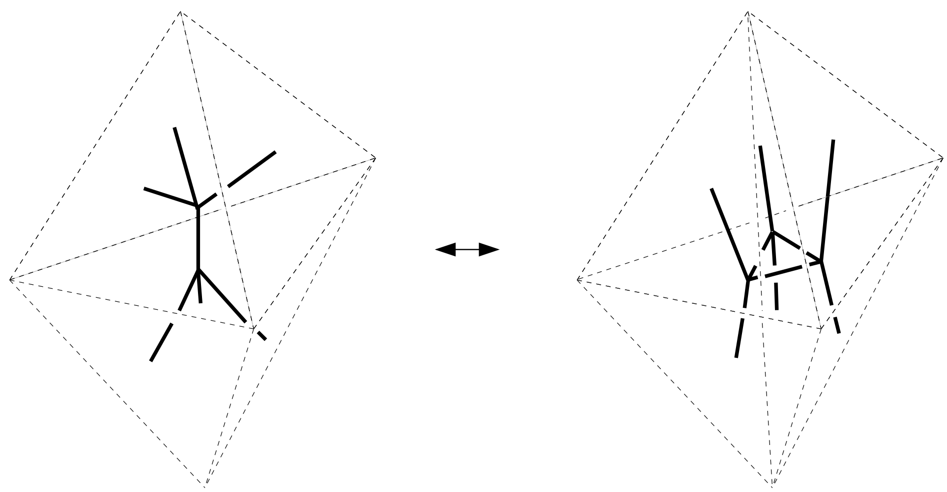
So far everything has been about space at a given time. What about *dynamics?* I'll describe a theory called the Barrett–Crane model, and some computer simulations of this model. For simplicity I'll discuss the *Riemannian* Barrett–Crane model, instead of the more realistic *Lorentzian* one.

In the Barrett–Crane model we assume space at any given time is built from tetrahedra, and the spin network lies in the 'dual 1-skeleton':

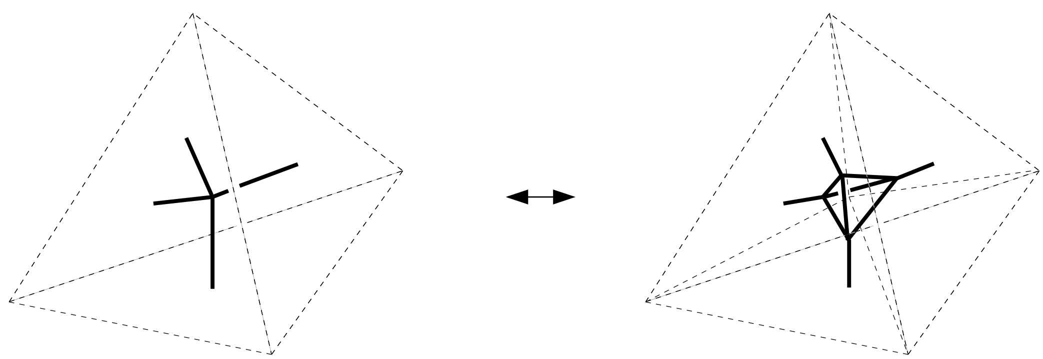


The spins $j_1, \ldots, j_4$ describe the areas of the triangles. Given these spins, the theory picks out a specific intertwining operator at the vertex.

Time evolution proceeds randomly by two moves: the **2-3 move**:

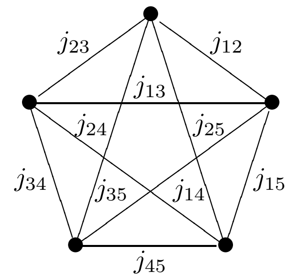


and the **1-4 move**:



Both these moves result from replacing the 'back' of a 4-simplex by the 'front'. Pachner's theorem says we can go between any two triangulations of a compact 3-manifold via these moves.

The spins on triangles unaffected by these moves don't change; the new spins are chosen randomly with an amplitude depending on all the spins involved. There are 10 of these spins, giving a spin network with one edge for each triangle in a 4-simplex:



The amplitude is called the **10*j* symbol**, and given by a certain integral:

$$\int_{(S^3)^5} \prod_{k<l} K_{j_{kl}}(\phi_{kl}) \; dx_1 \cdots dx_5.$$

Here the unit sphere $S^3 \subset \mathbb{R}^4$ is equipped with its rotation-invariant measure dx with total volume 1, ϕ_{kl} is the angle between the unit vectors x_k and x_l, and

$$K_j(\phi) = \frac{\sin(2j+1)\phi}{\sin\phi}$$

Problem: compute this integral efficiently! It's hard to do directly. Naive methods using representation theory take $O(j^9)$ time and $O(j^4)$ space. Christensen and Egan developed a clever method requiring only $O(j^5)$ time and $O(j^4)$ space. The code is available to all.

Some Sample Calculations

Consider a tiny spacetime: a 4-sphere triangulated with six 4-simplices (the boundary of a 5-simplex). What are the probabilities with which the triangles are labelled by various spins in the Barrett–Crane model?

Baez, Christensen, Halford and Tsang did a Monte Carlo calculation with spin cutoff $J = 50$ and half a billion iterations. Each iteration required computing six $10j$ symbols. We obtained the following results:

spin	frequency
0	69.548%
1/2	18.733%
1	6.2878%
3/2	2.5510%
2	1.1958%
5/2	.61995%
3	.34893%
7/2	.21243%
4	.13535%
9/2	.08989%
5	.06252%

Another question: *what is the expected area of a triangle in this spacetime?*

cutoff J	expected triangle area
0	0.000000
1/2	0.121987
1	0.210441
3/2	0.265911
2	0.302153
5/2	0.326524
15/2	0.381160
25/2	0.396701
50	0.399991
∞	0.400005

The results above are *exact* when the cutoff J is $\leq \frac{5}{2}$: we averaged over all labellings of the triangles in this spacetime by spins $\leq J$. This required summing 3.5 trillion products of six $10j$ symbols when $J = \frac{5}{2}$. The Beowulf cluster at UWO came in handy here. Results for higher cutoffs are approximate, obtained by a Monte Carlo calculation.

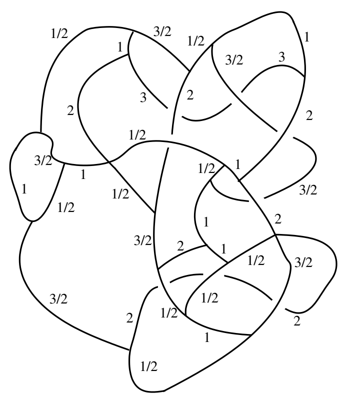
This result looks plausible, but we also obtained many results that physicists didn't expect!

Moral for physicists:

In a regime where analytical methods don't work well (yet), we need computer simulations to test our models.

Moral for computer scientists:

In loop quantum gravity, the geometry of space is built from qubits. Spacetime is like a parallel-processing quantum computer that constantly modifies its own topology.



Challenge for computer scientists:

Figure out how to simulate such systems more efficiently!

There are many concrete subgoals here, such as finding an efficient way to compute the Lorentzian $10j$ symbols.