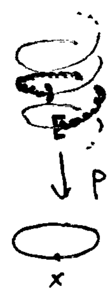
12 Jan 07
Derek

Homework #2

52.3 ?
53.1
53.5
Write your own proofs for Thm 53.1, 53.3

Restrictions of covering maps aren't necessarily covering maps:



p restricted to this part is surjective - it "covers" $S^1$ - but the point $x$ has no neighborhood that is evenly covered.

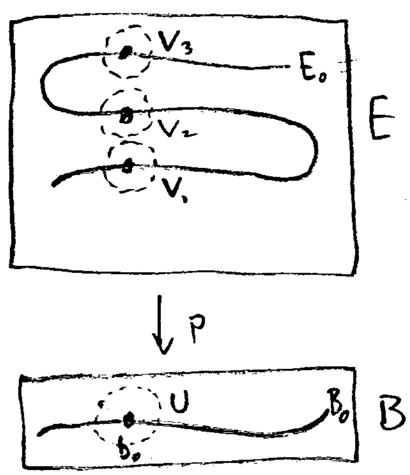
When is it a covering map? When we restrict it to the preimage of a subspace of B.

Thm 53.2 Let $p : E \to B$ be a covering map. If $B_0 \subset B$, let $E_0 = p^{-1}(B_0)$; then

$$p_0 = p\big|_{E_0} : E_0 \to B_0$$

is a covering map.

Pf. Pick $b_0 \in B_0 \subset B$, and $U \underset{op}{\subseteq} B$ st. $b_0 \in U$ and $U$ is evenly covered by $p$. Then $p^{-1}(U) = \coprod_\alpha V_\alpha$ with $p|_{V_\alpha} : V_\alpha \to U$ homeo for each $\alpha$.



Note that $p^{-1}(U \cap B_0) = \bigcup_{\alpha} V_\alpha \cap E_0$, where $V_\alpha \cap E_0$ are disjoint.
So the restriction

$$p|_{V_\alpha \cap E_0} : V_\alpha \cap E \to U \cap B$$

is homeo for each $\alpha$. Hence $U \cap B_0$ is an open nbhd of $b_0$ in $B_0$ that is evenly covered by $p|E_0$.

(Note: $p|_{V_\alpha \cap E_0} = (p|_{E_0})|_{V_\alpha \cap E_0}$)

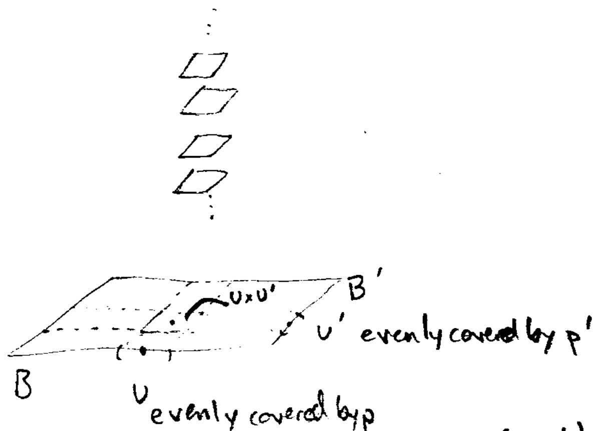
$\square$

<u>Thm 53.3</u>: The cartesian product of covering maps is a covering map.

Pf. Homework!

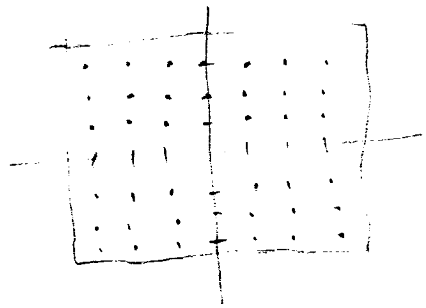
Here's the idea: Suppose $p: E \to B$, $p': E' \to B'$ are covering maps.

Show $p \times p' : E \times E' \to B \times B'$ is a covering map.

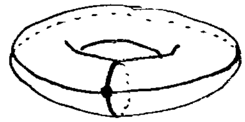



so $U \times U'$ evenly covered

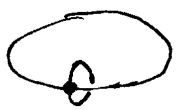
Ex ① Take $p, p'$ both to be usual covering $\mathbb{R} \to S^1$



$$\mathbb{R} \times \mathbb{R} \xrightarrow{p \times p'} S^1 \times S^1 = T^2$$



② Let $B_0 \subset T^2$ be this subspace

 $\subset$ 

$\parallel$

 $= S^1 \vee S^1 =$ "figure-eight space"

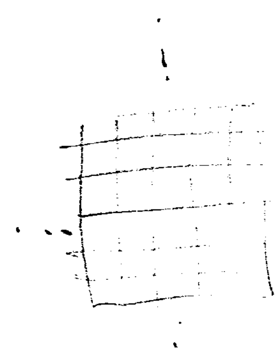
By Thm 53.2, if

$$p : \mathbb{R}^2 \to T^2$$

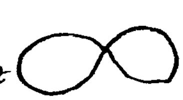
is the covering map in example 1, the restriction

$$p|_{E_0} : E_0 \to B_0$$

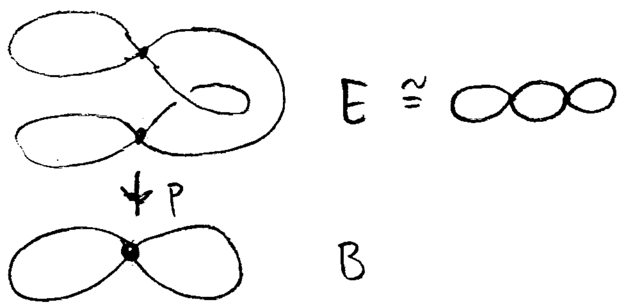
is a covering map, where $E_0 = p^{-1}(B_0)$



$E_0$ = grid subspace of $\mathbb{R}^2$

horz  vert

③ Another covering of the figure 8, this time twofold




<u>Thm 54.5:</u> For any basepoint $x_0 \in S^1$, $\pi_1(S^1, x_0) \cong \mathbb{Z}$.

This is "obvious" since any two loops with the same winding number should be homotopic, but proving it takes some work...

<u>Defn:</u> If $p: E \to B$ is any continuous map, a <u>lifting</u> of a map $f: X \to B$ is a map $\tilde{f}: X \to E$ st $p \circ \tilde{f} = f$, ie. s.t.

$$\begin{tikzcd} & E \arrow[d, "p"] \\ X \arrow[ur, dashed, "\tilde{f}"] \arrow[r, "f"'] & B \end{tikzcd}$$

commutes.

Liftings are especially nice when $p: E \to B$ is a covering map.