Definitions — give precise definitions for these terms.

1. Covering map.
2. Pushout.
3. Homotopy equivalence.
4. Retract.

Proofs, Examples and Counterexamples — Settle each question with a proof, an example, or a counterexample.

5. Let $S^1 \subset S^2$ be the ‘equator’ of the 2-sphere:

Is this subspace $S^1$ a retract of $S^2$ or not?

6. Let the Klein bottle $K$ be the quotient space of the square formed by identifying edges as shown:

Let $A \subseteq K$ be the subspace given by the edge labelled $A$. ($A$ is homeomorphic to a circle.) Is $A$ a retract of $K$?

7. Consider the situation in problem 8. Is the circle $A$ a deformation retract of $K$?

8. Is every subspace of a contractible space contractible?

Calculations — do the calculations, with just enough detail so I can see what you are doing.
9. Compute the fundamental group of $S^2$ with three points removed.

10. The Möbius strip $M$ is obtained as a quotient space of the square by identifying two edges as shown:

\[
\begin{array}{c}
\text{\rotatebox{90}{\textbullet}} \\
\text{\rotatebox{90}{\textbullet}} \\
\text{\rotatebox{-90}{\textbullet}} \\
\text{\rotatebox{-90}{\textbullet}} \\
\end{array}
\]

Compute the fundamental group of the Möbius strip.

11. Compute the fundamental group of the wedge product $(S^1, *) \lor (\mathbb{RP}^2, *)$.

**Proof** — Prove the following result.

12. Let $X$ be a space. Suppose $\gamma : [0, 1] \to X$ is a path from $x \in X$ to $y \in X$. Let $\bar{\gamma}$ be the reverse path:

$$
\bar{\gamma}(t) = \gamma(1 - t) \quad t \in [0, 1].
$$

Show that $\gamma \ast \bar{\gamma}$ is path homotopic to the constant path at $x$. 