MATH 205B
FINAL EXAM

Definitions — give precise definitions for these terms.

1. Covering map.

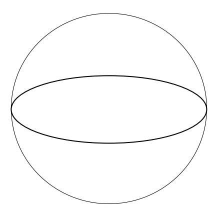
2. Pushout.

3. Homotopy equivalence.

4. Retract.

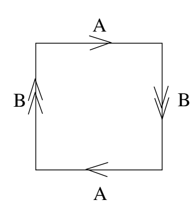
Proofs, Examples and Counterexamples — Settle each question with a proof, an example, or a counterexample.

5. Let $S^1 \subset S^2$ be the 'equator' of the 2-sphere:



Is this subspace S^1 a retract of S^2 or not?

6. Let the Klein bottle K be the quotient space of the square formed by identifying edges as shown:



Let $A \subseteq K$ be the subspace given by the edge labelled A. (A is homeomorphic to a circle.) Is A a retract of K?

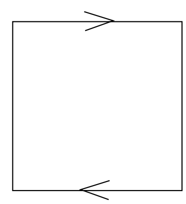
7. Consider the situation in problem 8. Is the circle A a deformation retract of K?

8. Is every subspace of a contractible space contractible?

Calculations — do the calculations, with just enough detail so I can see what you are doing.

9. Compute the fundamental group of S^2 with three points removed.

10. The Möbius strip M is obtained as a quotient space of the square by identifying two edges as shown:



Compute the fundamental group of the Möbius strip.

11. Compute the fundamental group of the wedge product $(S^1, *) \vee (\mathbb{R}\mathrm{P}^2, *)$.

Proof — Prove the following result.

12. Let X be a space. Suppose $\gamma\colon [0,1] \to X$ is a path from $x \in X$ to $y \in X$. Let $\overline{\gamma}$ be the reverse path:

$$\overline{\gamma}(t) = \gamma(1-t) \qquad\qquad t \in [0,1].$$

Show that $\gamma * \overline{\gamma}$ is path homotopic to the constant path at x.