MATH 205B
REVIEW HOMEWORK

Definitions — give precise definitions for these terms.

1. Path homotopy.

2. Functor.

3. Covering space.

4. Deformation retract.

Theorems — give precise statements of these theorems.

5. Brouwer fixed-point theorem.

6. Seifert-van Kampen theorem.

Proofs, Examples and Counterexamples — Settle each question with a proof, an example, or a counterexample.

7. Let $A \subset \mathbb{R}^2$ be the figure-8 space, thought of as a subset of the plane in the obvious way, as shown here:

$$8$$

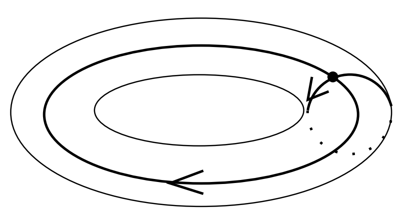
Is A a retract of $\mathbb{R}^2$ or not? Either describe a retraction or prove none exists.

8. Let $A \subset \mathbb{R}^2$ be the figure-8 space, as before. Let $B \subset A$ be the top half of the figure-8 space, which is homeomorphic to a circle. Is B a retract of A? Either describe a retraction or prove none exists.

9. Let A and B be as above. Is B a deformation retract of A? Either describe a deformation retraction or prove none exists.

10. Is the torus homeomorphic to the Klein bottle? Either describe a homeomorphism or prove none exists.

11. Is there a path homotopy between these loops in the torus T^2?



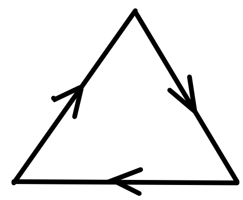
Either describe a path homotopy or prove none exists.

Calculations — do the calculations, with just enough detail so I can see what you are doing.

12. Compute the fundamental group of $(\mathbb{R}P^2, *) \vee (\mathbb{R}P^2, *)$. Is this group finite or infinite?

13. Compute the fundamental group of the two-holed torus with a point removed.

14. Compute the fundamental group of the quotient space formed by identifying all three edges of a solid triangle as shown:

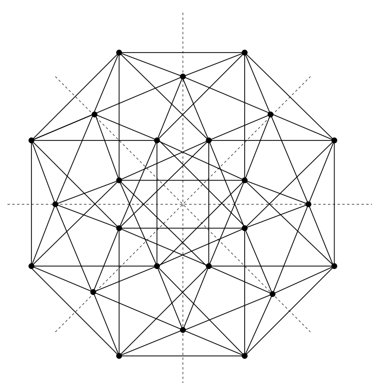


15. Find a pointed space whose fundamental group is $\mathbb{Z}/5$.

Proofs — Prove the following results.

16. Prove that $\mathbb{R}^n$ is contractible.

17. Suppose you are given the fact that any continuous vector field on the closed unit ball $D^n \subseteq \mathbb{R}^n$ points directly inwards somewhere on the boundary of the ball. Using this, prove that any map $f\colon D^n \to D^n$ has a fixed point.



This is the *24-cell*, a 4-dimensional polytope that's a kind of hybrid of the hypercube and the cross-polytope!