Quantization of Area: The Plot Thickens

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One of the key predictions of loop quantum gravity is that the area of a surface can only take on a discrete spectrum of values. In particular, there is a smallest nonzero area that a surface can have. We can call this the 'quantum of area', so long as we bear in mind that not all areas are integer multiples of this one — at least, not in the most popular version of the theory.

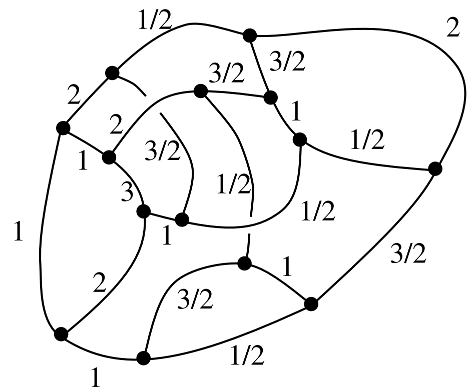
So far, calculations working strictly within the framework of loop quantum gravity have been unable to determine the quantum of area. But now, thanks to work of Olaf Dreyer [1] and Luboš Motl [2], two very different methods of calculating the quantum of area have been shown to give the same answer: $4 \ln 3$ times the Planck area. Both methods use semiclassical ideas from outside loop quantum gravity. The first uses Hawking's formula for the entropy of a black hole, while the second uses a formula for the frequencies of highly damped vibrational modes of a classical black hole. It is still completely mysterious why they give the same answer. It could be a misleading coincidence, or it could be an important clue. In any event, the story is well worth telling.

The importance of *area* in quantum gravity has been obvious ever since the early days of black hole thermodynamics. In 1973, Bekenstein [3] argued that the entropy of a black hole was proportional to its area. By 1975, Hawking [4] was able to determine the constant of proportionality, arriving at the famous formula

$$S = A/4$$

in units where $\hbar = c = G = k = 1$. Understanding this formula more deeply has been a challenge ever since.

Things took a new turn around 1995, when Rovelli and Smolin [5] showed that in loop quantum gravity, area is quantized. The geometry of space is described using 'spin networks', which are roughly graphs with edges labelled by spins:



Any surface gets its area from spin network edges that puncture it, and an edge labelled by the spin j contributes an area of $8\pi\gamma\sqrt{j(j+1)}$, where γ is a dimensionless quantity called the Barbero–Immirzi parameter [6, 7].

Given this, it was tempting to attribute the entropy of a black hole to microstates of its event horizon, and to describe these in terms of spin network edges puncturing the horizon. After some pioneering work by Rovelli and Smolin, Krasnov [8] noticed that the horizon of a nonrotating black hole could be described using a field theory called Chern-Simons theory. He began working with Ashtekar, Corichi and myself on using this to compute the entropy of such a black hole.

By 1997 we felt we were getting somewhere, and we came out with a short paper outlining our approach [9]. While the details are technical [10], the final calculation is easy to describe. The geometry of the event horizon is described not only by a list of nonzero spins j_i labelling the spin network edges that puncture the horizon, but also by a list of numbers m_i which can range from $-j_i$ to j_i in integer steps. The intrinsic geometry of the horizon is flat except at the punctures, and the numbers m_i describes the angle deficit at each puncture. To count the total number of microstates of a black hole of area near A, we must therefore count all lists j_i, m_i for which

$$A \cong \sum_i 8\pi\gamma\sqrt{j_i(j_i+1)}.$$

This is a nice little math problem. It turns out that for a large black hole, the whopping majority of all microstates come from taking all the spins to be as small as possible. So, we can just count the microstates where all the spins j_i equal $1/2$. If there are n punctures, this gives

$$A \cong 4\pi\sqrt{3}\gamma n.$$

In a state like this, each number m_i can take just two values at each puncture. Thus if there are n punctures, there are 2^n microstates, and the black hole entropy is

$$S \cong \ln(2^n) \cong \frac{\ln 2}{4\pi\sqrt{3}\,\gamma}A.$$

In short, we see that entropy is indeed proportional to area, at least for large black holes. However, we only get Hawking's formula $S = A/4$ if we take the Barbero–Immirzi parameter to be

$$\gamma = \frac{\ln 2}{\pi\sqrt{3}}.$$

On the one hand this is good: it's a way to determine the Barbero–Immirzi parameter, and thus the quantum of area, which works out to

$$8\pi\gamma\sqrt{\tfrac{1}{2}(\tfrac{1}{2}+1)} = 4\ln 2.$$

This makes for a pretty picture in which almost all the spin network edges puncturing the event horizon carry one quantum of area and one qubit of information, as in Wheeler's 'it from bit' scenario [11]. One can also check that the same value of γ works for electrically charged black holes and black holes coupled to a dilaton field. On the other hand, it seems annoying that we can only determine the quantum of area with the help of Hawking's semiclassical calculation. The strange value of γ might also make us suspicious of this whole approach.

Meanwhile, as far back as 1974, Bekenstein [12] had argued that Schwarzschild black holes should have a discrete spectrum of evenly spaced areas. While this law does not hold in the loop quantum gravity description of black holes, it has some of the same consequences. For example, in 1986 Mukhanov [13] noted that with a law of this sort, the formula $S = A/4$ can only hold exactly if the nth area eigenstate has degeneracy k^n and the spacing between area eigenstates is $4\ln k$ for some number $k = 2, 3, 4,\ldots.$ He also gave a philosophical argument that the value $k = 2$ is preferred, since then the states in the nth energy level can be described using n qubits.

Many researchers have continued this line of thought in different ways, but in 1995, Hod [14] gave an remarkable argument in favor of $k = 3$. His idea was to determine the quantum of area by looking at the vibrational modes of a *classical* black hole! Hod argues that if classically a system can undergo periodic motion at some frequency ω, then in the quantum theory it can emit or absorb quanta of radiation with the corresponding energy. But the energy of a Schwarzschild black hole is just its mass, and this is related to the area of its event horizon by

$$A = 16\pi M^2,$$

so when a black hole absorbs one quantum of radiation its area should change by

$$\Delta A = 32\pi M \Delta M = 32\pi M\omega.$$

And now for the miracle! A nonrotating black hole will exhibit damped oscillations when you perturb it momentarily in any way, and there are different vibrational modes called quasinormal modes, each with its own characteristic frequency and damping. In 1993, Nollert [15] used computer calculations to show that in the limit of large damping, the frequency of these modes approaches a specific number depending only on the mass of the black hole:

$$\omega \cong 0.04371235/M.$$

Plugging this into the previous formula, Hod obtained the quantum of area

$$\Delta A = 4.394444$$

and noticed that this was extremely close to $4 \ln 3 = 4.394449$. On the basis of this, he daringly concluded that $k = 3$.

Our story now catches up with recent developments. In November 2002, Dreyer [1] found an ingenious way to reconcile Hod's result with the loop quantum gravity calculation. The calculation due to Ashtekar *et al* used a version of loop quantum gravity where the gauge group is SU(2). This is why so many formulas resemble those familiar from the quantum mechanics of angular momentum, and this is why the smallest nonzero area comes from a spin network edge labelled by the smallest nonzero spin: $j = 1/2$. But there is also a version of loop quantum gravity with gauge group SO(3), in which the smallest nonzero spin is $j = 1$. Dreyer observed out that if we repeat the black hole entropy calculation using this SO(3) theory, we get a quantum of area that matches Hod's result! One can easily check this by redoing the calculation sketched earlier, replacing $j = 1/2$ by $j = 1$. One finds a new value of the Barbero–Immirzi parameter:

$$\gamma = \frac{\ln 3}{2\pi\sqrt{2}},$$

and obtains $4 \ln 3$ as the new quantum of area. But ultimately, all that really matters is that when $j = 1$ there are 3 spin states instead of 2. Thus each quantum of area carries a 'trit' of information instead of a bit, which is why Dreyer obtains $k = 3$.

With the appearance of Dreyer's paper, the suspense became almost unbearable. After all, Hod's observation relied on numerical calculations, so the very next digit of his number might fail to match that of $4 \ln 3$. Luckily, in December 2002, Motl [2] showed that the match is exact! He used an ingenious analysis of Nollert's continued fraction expansion for the asymptotic frequencies of quasinormal modes.

While exciting, these developments raise even more questions than they answer. Why should SO(3) loop quantum gravity be the right theory to use? After all, it seems impossible to couple spin-1/2 particles to this version of the theory. Corichi has sketched a way out of this problem [16], but much work remains to see whether his proposal is feasible. Can we turn Hod's argument from a heuristic into something a bit more rigorous? He cites Bohr's correspondence principle in this form: "transition frequencies at large quantum numbers should equal classical oscillation frequences." However, this differs significantly from the idea behind Bohr–Sommerfeld quantization, and it is also unclear why we should apply it only to the *asymptotic* frequencies of highly damped quasinormal modes. Can the mysterious agreement between SO(3) loop quantum gravity and Hod's calculation be extended to rotating black holes? Here a new paper by Hod makes some interesting progress [17]. Can it be extended to black holes in higher dimensions? Here Motl's new work with Neitzke gives some enigmatic clues [18]. Stay tuned for further developments.

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