Classical Mechanics, Lecture 1

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1 A Tiny Taste of the History of Mechanics

These are some incredibly sketchy notes, designed to convey just a tiny bit of the magnificent history of mechanics, from Aristotle to Newton. I've left out huge amounts of important and interesting stuff.

Aristarchus of Samos

Aristarchus (310 BC - ∼240 BC) argued that the Earth orbited the Sun. In his book *On the Sizes and Distances of the Sun and Moon* he calculated the relative distances of the Sun and Moon by noting the angle between the Sun and Moon when the Moon was half full. His logic was correct, but his measurement of the angle was wrong (87 degrees instead of 89.5), so he concluded that the Sun was 20 times farther than the Moon, when it's actually about 390 times farther.

Archimedes later wrote:

> You King Gelon are aware the 'universe' is the name given by most astronomers to the sphere the centre of which is the centre of the earth, while its radius is equal to the straight line between the centre of the sun and the centre of the earth. This is the common account as you have heard from astronomers. But Aristarchus has brought out a book consisting of certain hypotheses, wherein it appears, as a consequence of the assumptions made, that the universe is many times greater than the 'universe' just mentioned. His hypotheses are that the fixed stars and the sun remain unmoved, that the earth revolves about the sun on the circumference of a circle, the sun lying in the middle of the orbit, and that the sphere of fixed stars, situated about the same centre as the sun, is so great that the circle in which he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the centre of the sphere bears to its surface.

Archimedes complained that the last sentence was mathematically meaningless. Recent commentators have suggested that Aristarchus was tyring to say the sphere of fixed stars was *infinite* in size!

Eratosthenes

Eratosthenes measured the circumference of the Earth to be 252,000 stadia by comparing shadows at noon in two cities at different latitudes. Unfortunately we don't know exactly how long his 'stadia' are! So, all we know is that his figure is within 20% of the right answer (about 40,000 kilometers).

Aristotle

Aristotle wrote his *Physics* as lecture notes sometime around 350 BC. His work was more philosophical than quantitative in nature, unlike the previous authors listed. He adopted an Earth-centered universe.

Some basic principles: All that moves is moved by something else. Action at a distance is inconceivable: the mover must always be connected to the moved.

What about falling bodies? This proved a bit embarrassing, since it seems to violate the above principles.

Aristotle gives no formulas, but at points he seems to suggest that velocity is proportional to force divided by "resistance":

$$v \propto F/R$$

For falling bodies this might mean

$$v \propto m/R$$

where m is the body's mass. This is actually *true* for a body falling at terminal velocity in air or some other medium with friction, but it doesn't address the problem of how falling bodies *accelerate*.

Aristotle believed that the four elements (earth, water, air and fire) each sought their own proper level, so earth fell towards the center of the universe, with water above that, then air, then fire. He believed that the stars and planets were made of a different substance than terrestrial matter ('aether', later called 'quintessence', meaning 'fifth element'). So, it probably never occurred to him to find a unified theory of motion applying equally to a falling rock and the moon. This was Newton's huge achievement.

Archimedes

Archimedes (287 BC - 212 BC) did amazing work on statics — and as we now know through recently discovered texts, he essentially invented the integral calculus.

Ptolemy

Ptolemy seems to have lived in Alexandria, 83 – 161 AD. In his *Almagest* he described a geocentric solar system with epicycles to compute the motions of the Sun, Moon, and 5 visible planets. It's extremely accurate, and makes use of some very interesting mathematics. Most people who make fun of Ptolemy's epicycles are complete idiots compared to Ptolemy.

Dark Ages

When the Romans took over Greece a long process of scientific decline began, nicely discussed in Lucio Russo's *The Forgotten Revolution: How Science Was Born in 300 BC and Why It Had to Be Reborn*. Most Greek texts were lost completely; the ones that survived did so through a highly complex process of repeated translation, discussed in Scott Montgomery's *Science in Translation: Movements of Knowledge through Cultures and Time*. A quote from the review by John Stachel:

> Perhaps the best of the book's many delightful challenges to conventional wisdom comes in the first section on the translations of Greek science. Here we learn why it is ridiculous to use a phrase like "the Renaissance recovery of the Greek classics"; that in fact the Renaissance recovered very little from the original Greek and that it was long before the Renaissance that Aristotle and Ptolemy, to name the two most important examples, were finally translated into Latin. What the Renaissance did was to create a myth by eliminating all the intermediate steps in the transmission. To assume that Greek was translated into Arabic "still essentially erases centuries of history" (p. 93). What was translated into Arabic was usually Syriac, and the translators were neither Arabs (as the great Muslim historian Ibn Khaldun admitted) nor Muslims. The real story involves Sanskrit compilers of ancient Babylonian astronomy, Nestorian Christian Syriac-speaking scholars of Greek in the Persian city of Jundishapur, and Arabic- and Pahlavi-speaking Muslim scholars of Syriac, including the Nestorian Hunayn Ibn Ishak (809-873) of Baghdad, "the greatest of all translators during this era" (p. 98).

So, it's important to remember that the 'Dark Ages' were dark only in Western Europe ('Christendom'), and that meantime Muslim scholars were making progress in astronomy, mathematics and so on. But, when the West finally got going, it did some wonderful things.

Etienne Tempier, Bishop of Paris

In the Middle Ages almost all Western European scholars were monks. The works of Aristotle were introduced around 1200 when they were imported via Arab sources, for example from Andalusia (now southern Spain). As they spread through Europe, they caused conflict with accepted views (a blend of Christian theology and Plato's philosophy).

In 1277 the Bishop of Paris condemned a list of 219 theses, including some related to physics. Here are some of the condemned theses:

- 66. That God is unable to to impart rectilinear uniform motion to the heavens.
- 102. That nothing happens by chance, but everything comes about by necessity, and that all the things that will exist in the future will exist by necessity...

Also: if one thing affects another, the second must also affect the first!

William Occam

Occam (1288-1347) was a Franciscan friar famous for his 'razor'; he also believed that in the absence of resistance motion would continue indefinitely.

Nicole Oresme

Oresme (1323-1382) was perhaps the first to draw pictures resembling graphs, which plot the change of some quantity (or 'form') as a function of time — though not on a rectilinear grid of 'graph paper'. This is very important because it's a step towards the later idea that time is like space.

In his book *Latitude of Forms* he studied many ways one quantity could vary as a function of another, including 'uniformly difform' quantities, i.e. those that change at a constant rate. Using these charts he showed that an object whose speed was 'uniformly difform' would move a distance from time t_1 to time t_2 equal to $v_{mean}(t_2 - t_1)$ where v_{mean} is the object's speed at a time *halfway between* t_1 *and* t_2. In modern language: if the acceleration

$$v'(t) = a$$

is constant, the change of position is

$$\int_{t_1}^{t_2} v(t)\, dt = \frac{1}{2} a (t_2 - t_1)^2$$

Later people including (but not only) Galileo applied this idea to falling objects.

Oresme also proved the divergence of the harmonic series!

Nicolaus Copernicus

In 1543, in his *De Revolutionibus Orbium Coelestium*, Copernicus rejects Ptolemy's model of the solar system and reverted to an earlier Greek model in which the Earth goes around the Sun and all orbits are perfectly circular. This makes predictions much less accurate than Ptolemy's!

Johannes Kepler

In 1596, Kepler published his *Mysterium Cosmographicum*, which adopted a Copernican heliocentric cosmology and attempted to explain the radii of the planet's orbits in terms of nested Platonic solids.

Later he spent years analyzing accurate data collected with his boss Tycho Brahe, and came up with a system where planets moved along circular orbits *not quite centered at the Sun*. (The off-center circle idea was already familiar to Ptolemy and called in Latin a *punctum aequans*.) However, he discovered slight discrepancies in the orbit of Mars (just 8 minutes, a minute being a 60th of a degree) which eventually led him to discard this system.

In the years that followed, he realized first that the planets could not move with constant speed around their orbits, and then that the orbits should be *ellipses*. In his 1609 book *Astronomia Nova* he formulated these laws:

1. Each planet moves along an ellipse with the Sun at one focus.
2. The vector from the Sun to the planet sweeps out equal areas in equal times.
3. The ratio of the squares of the periods of two planets is equal to the ratio of the cubes of their semimajor axes.

Perhaps even more importantly, we see in Kepler's work these new features, listed by E. J. Dijksterhuis in his magnificent book *The Mechanization of the World Picture: Pythagoras to Newton*:

1. Rejection of all arguments which are based solely on tradition and authority.
2. Independence of scientific inquiry of all philosophical and theological tenets.
3. Constant application of the mathematical mode of thought in the formulation and elaboration of hypothesis.
4. Rigorous verification of the results deduced by the latter by means of an empiricism raised to the highest degree of accuracy

Galileo Galilei

In 1632 Galileo wrote a book on Copernican astronomy versus Ptolemaic astronomy, *Dialogue Concerning the Two Chief World Systems*. Among other things, he formulated the principle of relativity of motion to explain why we wouldn't fall off a moving Earth:

> Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction.
>
> When you have observed all these things carefully (though doubtless when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still.

He also argued that discounting wind resistance, a falling object would fall at a constant acceleration independent of its mass. Using the same geometrical argument as Oresme and others, he saw that this meant it would fall a distance proportional to t^2.

Isaac Newton

Isaac Newton unified the work of Galileo and others on 'terrestrial mechanics' (falling bodies) with the work of Kepler and others on 'celestial mechanics' (the motion of planets). In his *Philosophiae Naturalis Principia Mathematica*, published in 1687 after enormous delays, he formulated three laws of motion:

- *Lex I: Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.*

  Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed.

- *Lex II: Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur.*

  The rate of change of momentum of a body is proportional to the resultant force acting on the body and is in the same direction.

- *Lex III: Actioni contrariam semper et fqualem esse reactionem: sive corporum duorum actiones in se mutuo semper esse fquales et in partes contrarias dirigi.*

  All forces occur in pairs, and these two forces are equal in magnitude and opposite in direction.

Note the third law is one of the doctrines condemned by the Bishop of Paris.

Oversimplifying enormously, one can say that a key step here was going from a first-order differential equation (trying to explain velocity) to a second-order one (trying to explain acceleration). Newton's second law can be formulated as a differential equation

$$F = ma$$

or

$$F = m\frac{d^2q}{dt^2}$$

where m is the body's mass, $q\colon \mathbb{R} \to \mathbb{R}^3$ is its position as a function of time, and F is the force upon it (typically some function of q, $\frac{dq}{dt}$, and perhaps t as well. This formulation is anachronistic since Newton didn't use *vectors*, but he did invent the differential and integral calculus. (So did Leibniz: Newton would write $\dot{q}$ while Leibniz wrote $\frac{dq}{dt}$.)

Newton's colleague Robert Hooke suggested had that the gravitational force exerted by one body on another was inversely proportional to the square of the distance between them. In an amazing *tour de force*, Newton was able to derive Kepler's three laws from this assumption. In a guided homework exercise we will derive the first. Then you'll see how smart Newton must have been!