Classical Mechanics Homework

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John Baez homework by C. Pro

Newton's 2nd law and constant acceleration.

A particle near the Earth's surface feeling only the force of gravity. This force is approximately independent of time and position:

$$F(t) = (0, 0, -mg).$$

Here we are in $\mathbb{R}^3$ and g is the downwards acceleration due to gravity - approximately 9.8 meters/second^2.

Solve Newton's second law $F = ma$ for $q(t) \in \mathbb{R}^3$ for this $F(t)$ - find $q(t)$ in terms of the initial position $q(0)$ and $\dot{q}(0)$. Hint: the path it traces out is a parabola.

Solution: Write $q(t) = (q_1(t), q_2(t), q_3(t))$ where $q_i \colon \mathbb{R} \to \mathbb{R}$. From Newton's 2nd law, we have $\ddot{q}_1(t) = \ddot{q}_2(t) = 0$ and $\ddot{q}_3(t) = -g$. It follows that $q_i(t) = q_i(0) + \dot{q}_i(0)t$ for $i = 1, 2$ and $q_3(t) = q_3(0) + \dot{q}_3(0)t - gt^2/2$. Therefore,

$$q(t) = (q_1(0) + \dot{q}_1(0)t, q_2(0) + \dot{q}_2(0)t, q_3(0) + \dot{q}_3(0)t - \frac{g}{2}t^2).$$

Newton's 2nd law and linear acceleration.

Consider a mass m attached to an ideal spring. Define the position q to be the distance of the mass relative to the equlibrium position (i.e., when the system is at rest). By Hooke's law, the force in this situation is approximately given by

$$F(t) = -kq(t),$$

where $k \geq 0$ is the spring constant.

Solve Newton's second law for $q(t) \in \mathbb{R}$ in terms of m, k, $q(0)$, $\dot{q}(0)$ and find the period P of the oscillation and the frequency ω, where $\omega = \frac{2\pi}{P}$ Hint: It oscillates!

Solution: Newton's second law says

$$\ddot{q}(t) + \frac{k}{m}q(t) = 0$$

which has a general solution of

$$q(t) = A\cos\left(\sqrt{\frac{k}{m}}t\right) + B\sin\left(\sqrt{\frac{k}{m}}t\right).$$

With our initial conditions, we have

$$q(t) = q(0)\cos\left(\sqrt{\frac{k}{m}}t\right) + \dot{q}(0)\sqrt{\frac{m}{k}}\sin\left(\sqrt{\frac{k}{m}}t\right).$$

Note the period and frequency are $P = 2\pi\sqrt{m/k}$ and $\omega = \sqrt{k/m}$, respectively.