Converging Realities: Toward a common philosophy of physics and mathematics, Princeton University Press, 2005, pp. 264 + xvii, Roland Omnès, Hardback, 0-691-11530-3, £18.95

Roland Omnès is a theoretical physicist known best to philosophers for his work on the foundations and interpretation of quantum mechanics. In this book he has daringly taken on the challenge of accounting for the fact that deep mathematical theories find themselves expressed in the laws of nature. Before we look at his thesis, I would like to make some meta-philosophical comments about people who are not professional philosophers constructing their own philosophical account of mathematics. Borrowing a notion from quantum mechanics, we might speak of a complementarity principle operating today in the philosophy of mathematics. We seem not to be able to have simultaneously work which is both scholarly and sufficiently robust to allow others to build upon it, while also capturing within a larger philosophical picture all the features mathematicians know to be crucial to the life of mathematics in its contemporary form. The situation is rather like a farmer who can either select a good piece of land, but must then take on too large an area to be able to prepare the soil properly, or who can prepare some ground well for future generations but only at the expense of being forced to pick a barren location. The first option is naturally the one we find scientists and mathematicians plumping for.

While the physicist or mathematician is careful and scholarly in their scientific publications, at times they seem to have little sense of the need for a similar rigour when they try their hand at philosophy of mathematics. Clearly close attention cannot be paid to all the details of a large picture if it is not to fill several volumes, but if an undergraduate writing an essay for me had said as Omnès does -

Some quasiempiricists were probably unaware of the growing influence of postmodern relativism, but Lakatos was definitely in the "postmodern" trend and his dialectic must be treated with caution. A common denominator of postmodernism is to reduce science to the status of an ordinary discourse, with no privilege over other types of discourse and no special right to claim privileged access to truth. (p. 220)

- red ink would have been very liberally applied. I find it hard to believe one could arrive at this assessment of Lakatos by reading any of his works. Presumably, Omnès has been misled by some secondary literature he has consulted. Other errors of this ilk are littered through the book, but they are counterbalanced by many fascinating ideas, which in my opinion ought to the kind of issues philosophers think about.

Are we condemned to suffer eternally from this complementarity principle? Will we have periodic intrusions from outsiders like Changeux and Connes' *Conversations on mind, matter and mathematics*, Hersh's *What is Mathematics, Really?*, or Mac Lane's *Mathematics: Form and Function*, each suggesting different agendas, while philosophers carry on wondering whether mathematics is just a part of second-order logic or whether some nuanced variant of Quine's indispensability argument is convincing. Returning to the agricultural analogy, the obvious solution to the problem faced by the farmer who has to tend too much land is to hire a team of workers. Clearly we need a huge collaborative effort. This is something Omnes recognises himself about his own program (p. 242). But consider how hard it will be to sign up researchers to a new cause in an environment where appraisal is achieved largely through the journal article. How much credit will you be given for seeming to be just correcting or filling in some detail of a larger schematic view of an outsider. Even those philosophers who have managed to tend more promising plots of land find it very hard to join forces, often keener instead to mark their considerable distance from big names in the field. Kitcher and Aspray's designation 'Maverick tradition', although it might be construed oxymoronic, seems apposite here (Kitcher & Aspray, 1988).

Returning to the book, little would be gained by my giving details of all the errors I found thoughout Omnès' book at the level of his description of other philosophical theses. Instead I shall concentrate on his positive thesis, which he names Physism. The full statement of this thesis is:

There are basic axioms for logic and mathematics. These axioms are laws of physics. They are recognized through two inseparable criteria: their fecundity in the construction of mathematics and their necessity for a statement of the law of physics. Their fecundity can be explained in view of the universality, subtlety, and richness of the laws: the basic axioms must be fecund enough to allow a statement of the laws in the language of mathematics.

Conversely, they generate every possible field of mathematics. New laws, new axioms, new fields, are possible and they may be discovered by further research.

Consistency is equally necessary in mathematics and in the laws of physics, which are inseparable. Consistency cannot be explained, but it stands as one of the two criteria of truth. The other one is experimental falsification of a mathematical proposition purporting to express a law of nature. (p. 215)

Clearly, many terms in this statement need clarifying.

What to me is the most important contribution of this book is to bring to centre stage a very important point. When wondering why our human mathematics is so well adapted to capture the physical world, it is not enough to say that our species has evolved in the natural world, that it has developed spatial and algorithmic tools to allow it to understand, predict and manipulate that world, and that all our current physical theories do is extend the range of these tools to the very small and the very large. Had Newtonian physics been the final word, this might have been plausible. We would just have been lucky that the laws governing the behaviour of bodies of our size continued at very different scales. But this is not the case. Omnès can speak of mathematics and physics growing together until well into the nineteenth century to produce *classical science*. This he characterises as follows:

A science is called "classical" when its concepts agree with the characteristics of the intuitive representation of reality, namely, uniqueness, location in ordinary space and time, continuity, separation of phenomena, causality, and a clear-cut distinction between the real and the virtual. (p. 21)

As we enter the twentieth century, however, all of these characteristics break down when physics discovers the quantum world. Indeed you have a devil of a job explaining how our classical reality can be seen to emerge from the quantum substratum. Omnès himself has made a very major contribution to this work, and gives a fascinating account of it here through the second part of the book. The key notion is *decoherence*. I am no philosopher of physics, but his account was very illuminating to me.

Returning to the story of the relationship between mathematics and physics, we noted that while a classical physics prevailed it was easy to see them as very thoroughly interwoven. But through the nineteenth century and early twentieth century each independently went through a series of crises prompting them to live out far less intimate lives, until a reunion was recognised to have taken place as late as the 1970s, at least in the West, allowing a new intimacy, growing every year in strength. An important mark of this relationship was physicist Edward Witten being awarded the Fields' Medal in 1990. Nowadays it is just a hop and a step from the mathematics of the Riemann hypothesis to the mathematics of string theory. But where does this leave us with the question why our current mathematics is so well adapted for physics? If a string theorist studies noncommutative tori, it's surely not from having dunked doughnuts in their coffee. I agree, then, with Omnès in rejecting Reuben Hersh's "Our mathematical ideas fit the world for the same reason that our lungs are suited to the atmosphere of this planet." (p. 165). Omnès' partial solution is to use physism to reduce two mysteries to one. We don't know why the laws of nature are mathematical, but that they are explains why mathematicians and physicists find themselves devising the same concepts.

One objection to raise to the thesis is to question whether mathematics and physics coincide. Are there not large tracts of mathematics which do not appear in science? Surely it would not be hard to think up plenty of mathematical results which one would expect never to grace a physics paper. Omnès' claim here is that "If one reduced mathematics to the unique and modest role of a language of the laws of physics, the present state of theoretical physics and simple consistency requirement would not appreciably modify the present corpus of mathematics." (pp. 88-89). The mathematical property of denseness might be used to nuance Omnès' position here. While the rationals have measure zero in the reals, every open set of reals contains a rational. Similarly, every substantial piece of mathematical theory might be found to contain a concept used in physics. To support this thesis would need a close consideration of the whole of mathematics. With limitations of space, Omnes does so for more challenging principles such as the Axiom of Choice.

I would agree that a place must be found for an account of the changing relationship between mathematics and physics over the past two centuries in any adequate philosophy. Indeed, I rather think that this account should be required to stretch further back to include the achievement of a Newtonianism which had come to the very unobvious conclusion that planets, cannon balls and pendula have much in common. One must avoid the danger in thinking that classical reality is quite so straightforward. But certainly we need to digest the lessons to be learnt from the quantum revolution. This can only take place within a very differently looking philosophy than we have been used to in the English-speaking world. Few philosophers of mathematics will be able to explain the interpretation of quantum mechanics due to David Bohm, but for Omnès the Bohmian position constitutes the first obvious objection to his thesis (pp. 232-5).

As the book's subtitle suggests, an alliance between philosophy of mathematics and philosophy of physics would need to be forged. This certainly strikes me as an attractive proposition. But something which would need to be overcome is the very commonly held view, but much less so amongst mathematical physicists, that the logical empiricists were more or less right to see mathematics as a part of logic, a body of tautologous if-then statements. Omnès gives an interesting sketch of the methodology of physics and mathematics in the Appendix to chapter 14. An overlooked part of this common methodology is what he calls the *conceptual stage*, in which the founding concepts of a theory are conceived. While philosophers of science have rescued this stage from being designated as the merely psychological context of discovery, the conceptual component of mathematical practice has been seriously neglected. I was glad to see that for Omnès, "The conceptual stage is also found in mathematics, where it is perhaps even more mysterious, like creativity itself." (p. 198)

To conclude, I recommend this book for its larger picture, rather than for its details. To be able to contribute to Omnès' program one would need, as he says, "a wide culture in mathematics, physics, history, and philosophy" (p. 242). How rare it surely must be to find an individual in which these are united.

Kitcher, P. and Aspray, W. (1988) 'An opinionated introduction', in Aspray, W. and Kitcher, P. Eds. (1988) *History and Philosophy of Modern Mathematics*, Minneapolis: University of Minnesota Press, pages 3-57.