QG F06 Homework 1
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1. *Evaluate the lambda-expression* $(\lambda fx.f(f(fx)))(\lambda gy.g(gy))(\lambda z.z+1)(0)$.

$$
\begin{aligned}
& (\lambda fx.f(f(fx)))(\lambda gy.g(gy))(\lambda z.z+1)(0) \\
= \; & (\lambda x.(\lambda gy.g(gy))((\lambda gy.g(gy))((\lambda gy.g(gy))x)))(\lambda z.z+1)(0) \\
= \; & (\lambda x.(\lambda gy.g(gy))((\lambda gy.g(gy))(\lambda y.x(xy))))(\lambda z.z+1)(0) \\
= \; & (\lambda x.(\lambda gy.g(gy))((\lambda gy.g(gy))(\lambda w.x(xw))))(\lambda z.z+1)(0) \\
= \; & (\lambda x.(\lambda gy.g(gy))(\lambda y.(\lambda w.x(xw))((\lambda w.x(xw))y)))(\lambda z.z+1)(0) \\
= \; & (\lambda x.(\lambda gy.g(gy))(\lambda y.(\lambda w.x(xw))(x(xy))))(\lambda z.z+1)(0) \\
= \; & (\lambda x.(\lambda gy.g(gy))(\lambda y.x(x(x(xy)))))(\lambda z.z+1)(0) \\
= \; & (\lambda x.(\lambda gy.g(gy))(\lambda w.x(x(x(xw)))))(\lambda z.z+1)(0) \\
= \; & (\lambda x.(\lambda y.(\lambda w.x(x(x(xw))))((\lambda w.x(x(x(xw))))y)))(\lambda z.z+1)(0) \\
= \; & (\lambda x.(\lambda y.(\lambda w.x(x(x(xw))))(x(x(x(xy))))))(\lambda z.z+1)(0) \\
= \; & (\lambda x.(\lambda y.x(x(x(x(x(x(xy)))))))(\lambda z.z+1)(0) \\
= \; & (\lambda y.(\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1) \\
& \quad ((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)y)))))))(0) \\
= \; & (\lambda y.(\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1) \\
& \quad ((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)(y+1)))))))(0) \\
= \; & (\lambda y.(\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)(y+2)))))))(0) \\
= \; & (\lambda y.(\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)(y+3))))))(0) \\
= \; & (\lambda y.(\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)(y+4)))))(0) \\
= \; & (\lambda y.(\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)(y+5))))(0) \\
= \; & (\lambda y.(\lambda z.z+1)((\lambda z.z+1)(y+6)))(0) \\
= \; & (\lambda y.(\lambda z.z+1)(y+7))(0) \\
= \; & (\lambda y.y+8)(0) \\
= \; & (0+8) \\
= \; & 8
\end{aligned}
$$

or, using Church numerals,

$$\begin{aligned}
&(\lambda fx.f(f(fx)))(\lambda gy.g(gy))(\lambda z.z+1)(0)\\
&= (3)(2)(inc)(0)\\
&= (8)(inc)(0)\\
&= 8
\end{aligned}$$

2. *Let $\omega = \lambda x.xx$. What is $\omega\omega$?*

β-reduction gives the same term we started with, so it's an infinite loop.