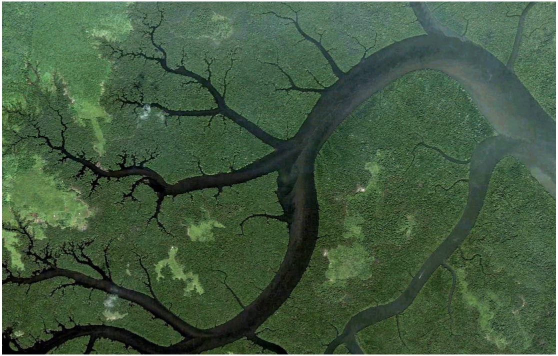
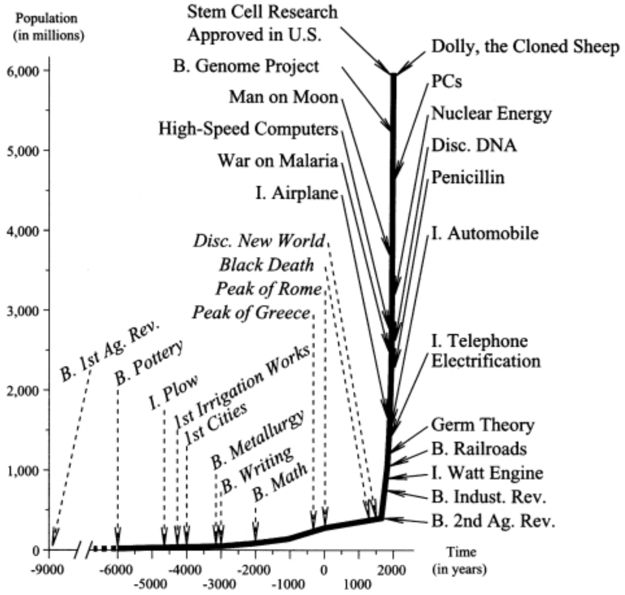
MATHEMATICS OF THE ENVIRONMENT

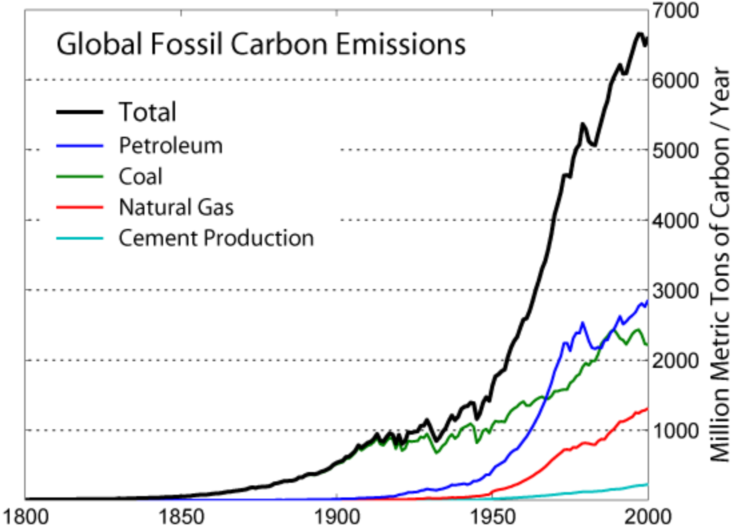


John Baez

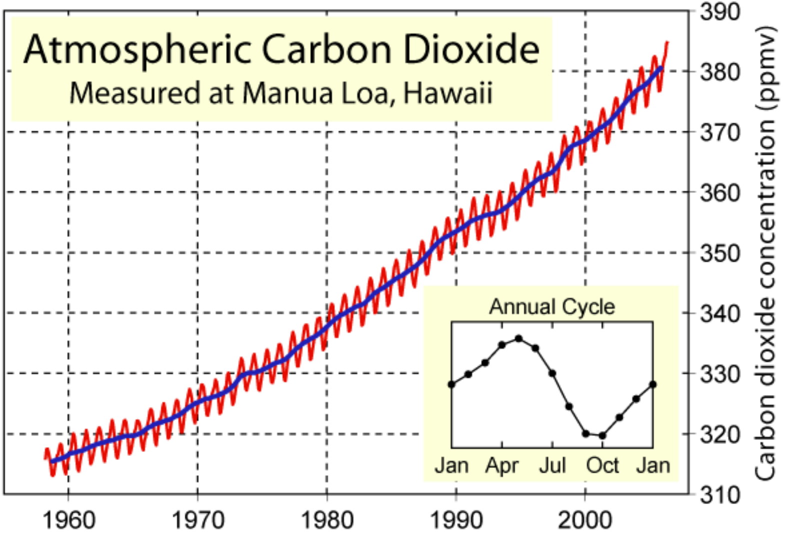
October 2, 2012



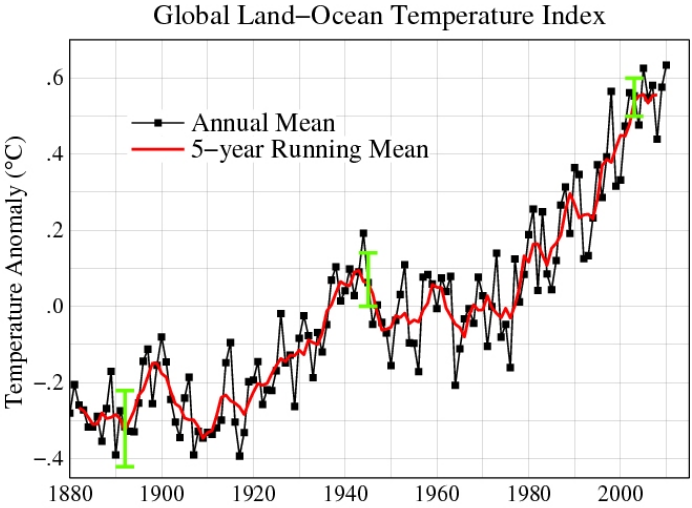
Robert Fogel - *The Escape from Hunger and Premature Death, 1700-2100*



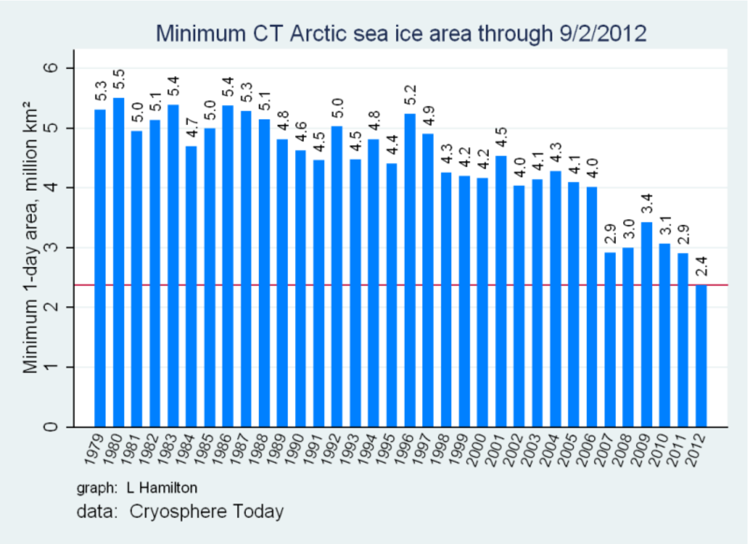
Global Warming Art



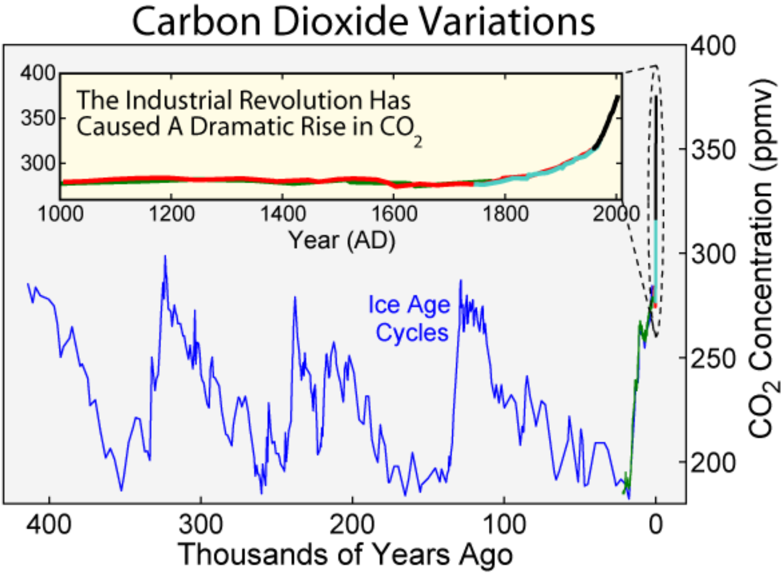
The Keeling Experiment — Global Warming Art



NASA Goddard Institute of Space Science



The Cryosphere Today



Antarctic ice cores and other data — Global Warming Art

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During this revolution, from 10,000 to 5,000 BC, we began to systematically exploit solar power by planting crops.

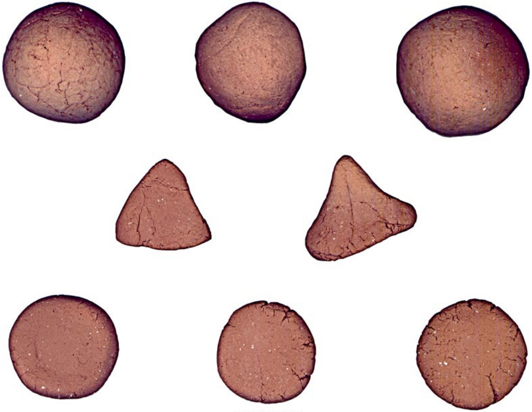
By now we use about 25% of all plant biomass grown worldwide! If this reaches 100% there will be, in some sense, no 'nature' separate from humanity.

Starting shortly after the end of the last ice age, the agricultural revolution led to:

- surplus grain production, and thus kingdoms and slavery.
- *astronomical mathematics* for social control and crop planning.
- *geometry* for measuring fields and storage containers.
- *written numbers* for commerce.

Consider the last...

Starting around 8,000 BC, in the Near East, people started using 'tokens' for contracts: little geometric clay figures that represented things like sheep, jars of oil, and amounts of grain.



MS 5067/1-8
Neolithic plain counting tokens. Near East, ca. 8000--3500 BC

The Schøyen Collection

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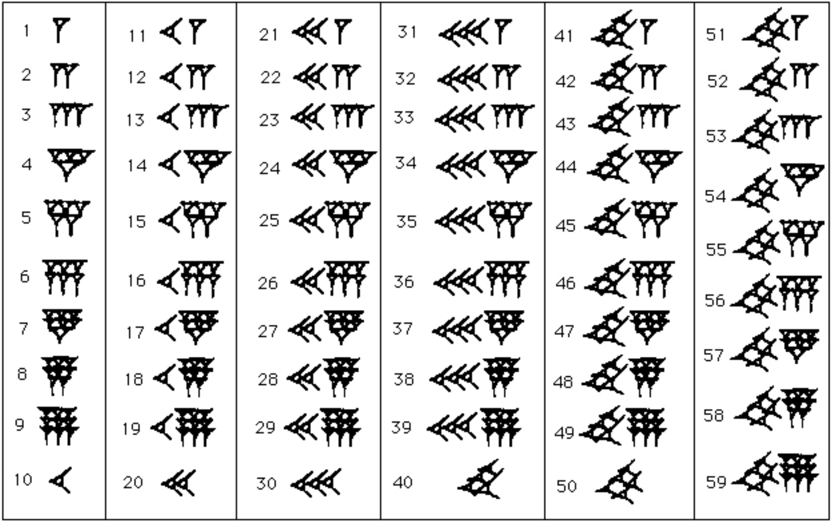
Eventually they gave up on the tokens. The marks on tablets then developed into the Babylonian number system! The transformation was complete by 3,000 BC.



J. J. O'Connor and E. F. Robertson, Babylonian Numerals

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By 1700 BC the Babylonians could compute $\sqrt{2}$ to 6 decimals:

$$1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} \approx 1.414213...$$

Yale Babylonian Collection, YBC7289

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Math may undergo a transformation just as big as it did in the Agricultural Revolution.

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Even better, these machines should spread without human intervention.

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For sophisticated ecotechnology we need to pay attention to what's already known—permaculture, systems ecology and so on. But better mathematics could help.

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Cao and Caldeira argue that if we double CO_2 in the air, *16% of land warming will be caused by this effect!*

But CO_2 also helps plants grow leaves. Bounoua *et al* say this effect would cool the land by 0.6 °C with doubled CO_2.

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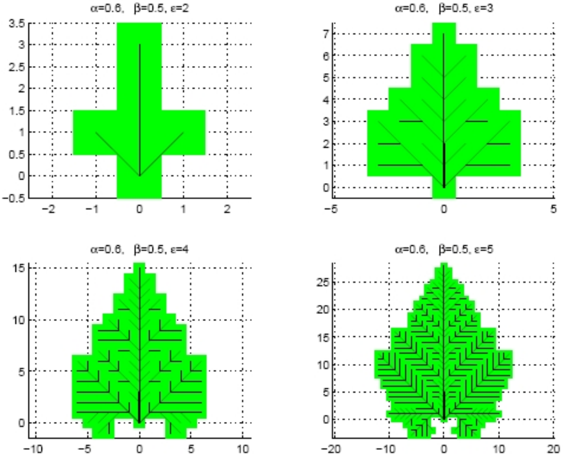
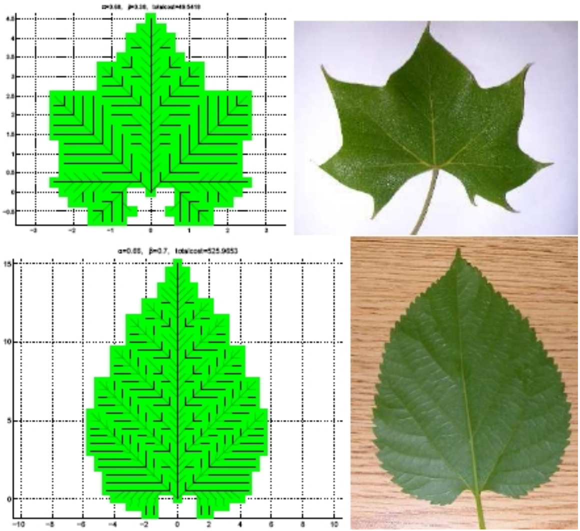
Yes! A mathematician at U.C. Davis, Qinglan Xia, has written a paper called *The Formation of a Tree Leaf*.

He models a leaf as a union of square cells centered on a grid, together with 'veins' forming a weighted directed graph from the centers of the cells to the root. The leaf grows new cells at the boundary while minimizing a certain cost function.



The cost function depends on two parameters. Changing these gives different leaf shapes:



Qinglan Xia's work is definitely math:

Lemma 3.8. *Suppose* (Ω, G) *is an* (ϵ, h) *leaf and* $(\mu, \Theta) = \phi_h(\Omega, G)$*. Then the total mass of the Radon measure is bounded above by*

$$\mathbf{M}(\mu) \leq \pi (R_\epsilon + h)^2$$

and the total variation of the vector measure Θ *is bounded by*

$$\mathbf{M}(\Theta) \leq \epsilon \pi^{2-\alpha} (R_\epsilon + h)^{4-2\alpha}.$$

Proof. Since $\Omega \subset B_{R_\epsilon}(O)$, the mass of μ is given by

$$\begin{aligned} \mathbf{M}(\mu) &= ||\Omega|| \, h^2 \\ &= \text{area}\left(\bigcup_{x \in \Omega} \left\{x + \left[-\frac{h}{2}, \frac{h}{2}\right] \times \left[-\frac{h}{2}, \frac{h}{2}\right]\right\}\right) \\ &\leq \text{area}\left(B_{R_\epsilon + h}(0)\right) = \pi (R_\epsilon + h)^2. \end{aligned}$$

Also, since $w(e) \leq ||\Omega|| \, h^2$ for each $e \in E(G)$, the total variation of Θ is given by

$$\begin{aligned} \mathbf{M}(\Theta) &= \sum_{e \in E(G)} w(e) \, length(e) \\ &\leq \left(||\Omega|| \, h^2\right)^{1-\alpha} \sum m_\beta(e^+) \, (w(e))^\alpha \, length(e) \end{aligned}$$

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It's just beginning to be born. At the Azimuth Project we're trying to help it along.