

MATHEMATICS IN THE 21ST CENTURY



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TOPOS INSTITUTE
MARCH 25, 2021**

Mathematics will continue to change dramatically in the 21st century, both for

- ▶ “endogenous” reasons — its internal logic

and

- ▶ “exogenous” reasons — coming from the outside world.

I'll talk about both — but especially how mathematics may be affected by, and affect, the Anthropocene.

In the early 21st century, mathematicians are trying to do some exciting things. We could all make our own list. Here's mine.

1. Embrace homotopical / higher-categorical thinking.

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homotopy type theory: doing math with new foundations based on homotopy types rather than sets.

full-fledged ∞ -categories: Grothendieck’s dream.

2. Understand quantum field theory & string theory.

Only the loftiest peaks, which reach above the clouds, are seen in the mathematical theories of today, and these splendid peaks are studied in isolation, because above the clouds they are isolated from one another. Still lost in the mist is the body of the range, with its quantum field theory bedrock and the great bulk of the mathematical treasures. So there is one rather safe, though perhaps seemingly provocative, prediction about twenty-first century mathematics: trying to come to grips with quantum field theory will be one of the main themes. — Edward Witten, 1998

3. Broaden our view of algebraic geometry.

The struggle to understand the “field with one element”, tropical algebra, etc., is pushing us to generalize the duality

$$\text{COMMUTATIVE ALGEBRA}^{\text{op}} \simeq \text{GEOMETRY}$$

that underlies algebraic geometry.

What sort of “space” has \mathbb{Z} as its ring of functions, really? Can understanding this help us prove the Riemann Hypothesis?

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Sheaves of commutative rings? Commutative algebra must work “locally”.

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Items 1–3 are all related, and they interact in exciting ways!

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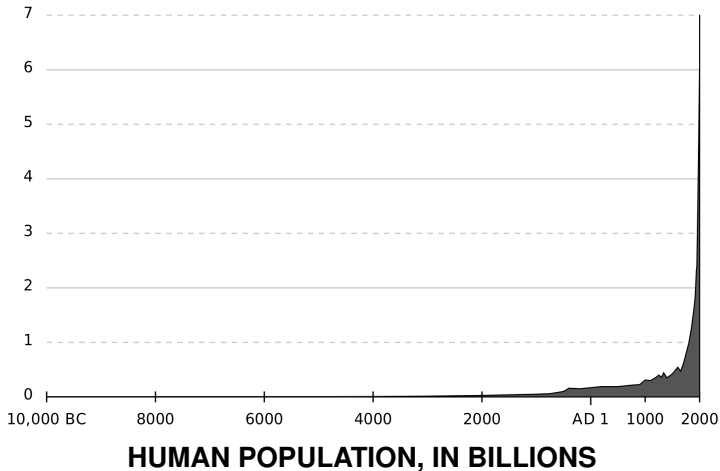
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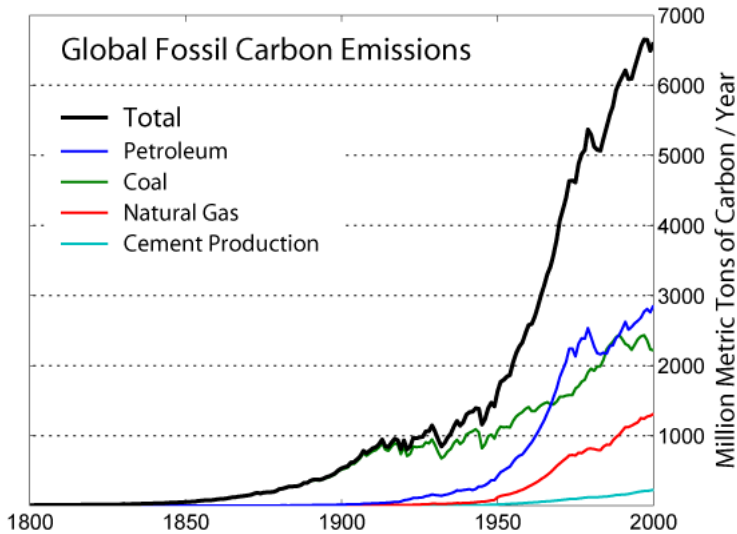
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One reason better math education is urgent: we're in a slow-burning crisis.

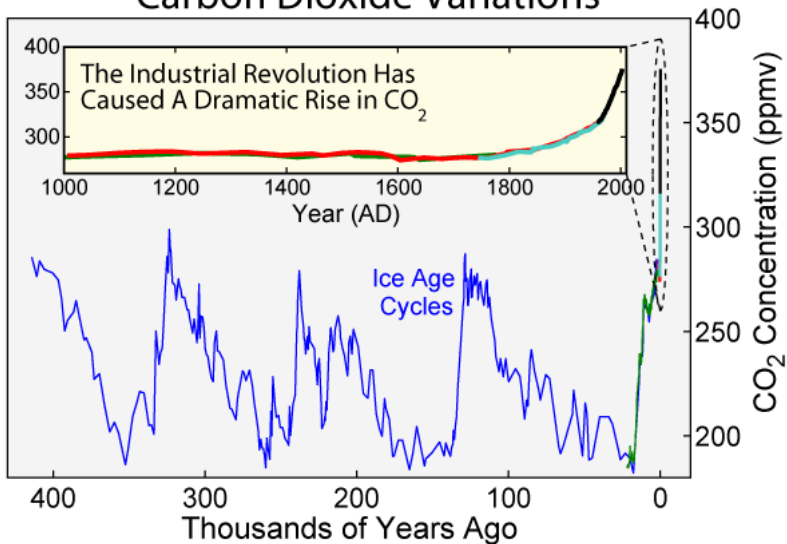
We have left the Holocene and entered a new epoch, the **Anthropocene**, when the biosphere is rapidly changing due to human activities. Global warming is just *part* of this process.



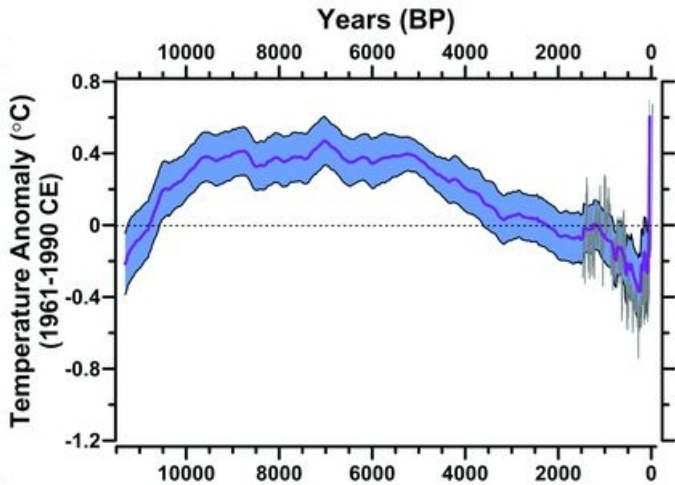


Carbon Dioxide Information Analysis Center

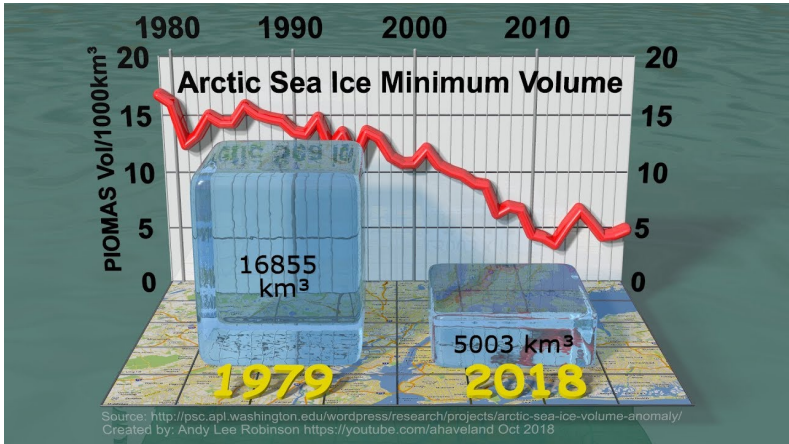
Carbon Dioxide Variations



Antarctic ice cores and other data — Global Warming Art



Reconstruction of temperature from 73 different records —
Marcott *et al.*



PIOMAS data, art by Andy Lee Robinson

San Francisco — September 9, 2020



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- ▶ 8-9 times as much phosphorus is flowing into oceans than the natural background rate.

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- ▶ **Consider having fewer children, or adopting.** In the US each child adds **~9400 tonnes** of CO₂ to the air.

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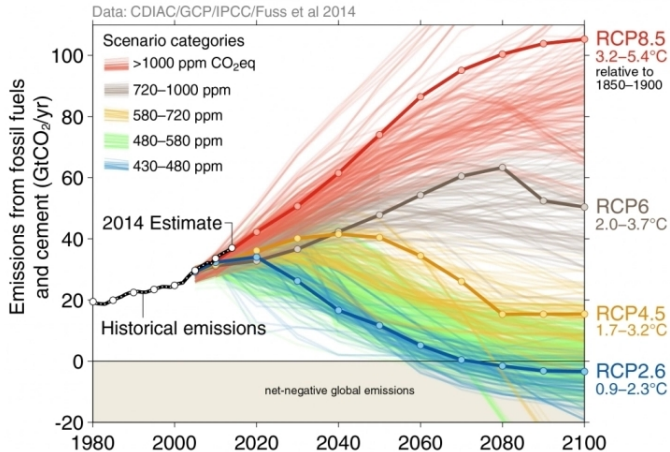
Pure mathematicians tend to feel helpless. “Is there any way I can help solve global warming while continuing to work on algebraic *K*-theory?”

But in fact there is a lot that even pure mathematicians can do.

- 1. Learn more about the problems and solutions — and talk to people about them!**

If you're watching this talk, you are probably better than average at dealing with quantitative data. For global warming, *the numbers matter*. Let me give you some.

To keep global warming below 2°C by 2100, it's likely that we will need *negative carbon emissions*:



But how much?

We are now putting **38 gigatonnes** of CO₂ into the air each year.

To keep global warming below 2°C by 2100, we will probably need to remove about **10 gigatonnes** of CO₂ from the air each year by 2050, and **double that** by 2100.

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For example: make plastics from CO₂ in the air? We make about **360 megatonnes** of plastic per year — not nearly enough to make a difference!

We could pull down over 10 gigatonnes of CO₂ per year this way:

- ▶ 1 gigatonne by planting trees,
- ▶ 1.5 gigatonnes by better forest management,
- ▶ 3 gigatonnes by better agricultural practices,
- ▶ 5.2 gigatonnes by biofuels with carbon capture.

This is not enough to cancel the 38 gigatonnes we're dumping into the air each year now, but *together with reducing emissions* it could be enough!

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- ▶ applied algebraic topology
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The **Topos Institute** promises to be a great place for this.

As an example: I'm using category theory to study networks. Why?

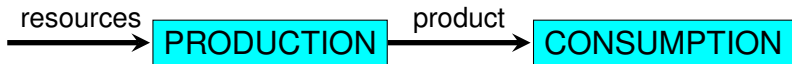
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In general, we have been pretending that:

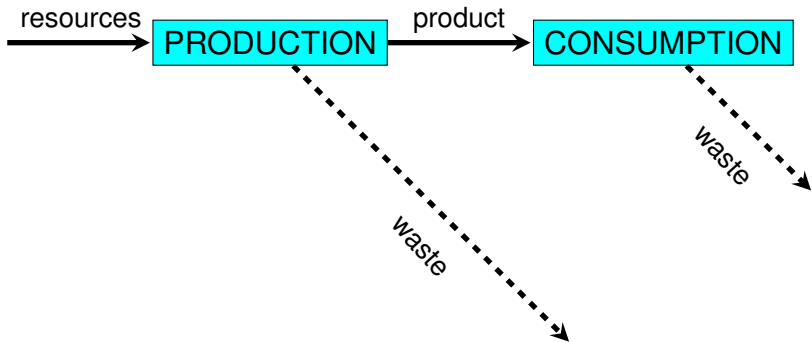
- ▶ the Earth is essentially infinite
- ▶ our effect on the biosphere is negligible
- ▶ exponential growth is a normal condition.

Acting as if these are true inevitably brings us to a point where they *stop* being true.

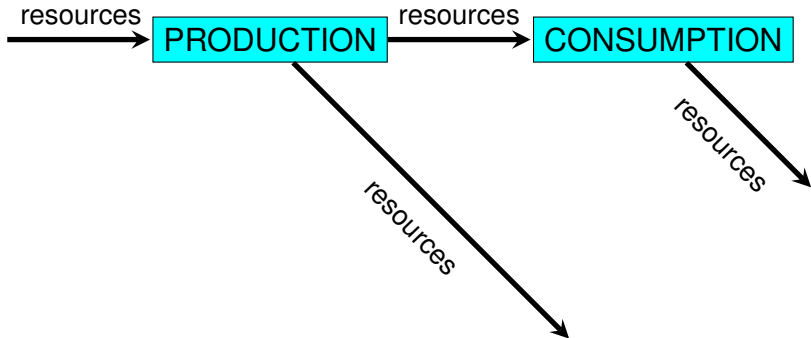
Our simplified picture:



Our simplified picture neglects “waste”, until it causes trouble:



In nature there is no waste, only more resources:



We need to bring nature into our economics:

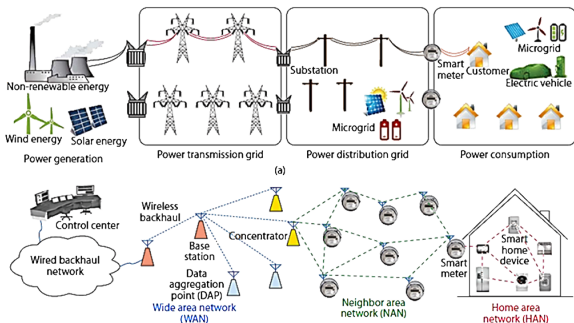
- ▶ *The Economics of Biodiversity: The Dasgupta Review.*

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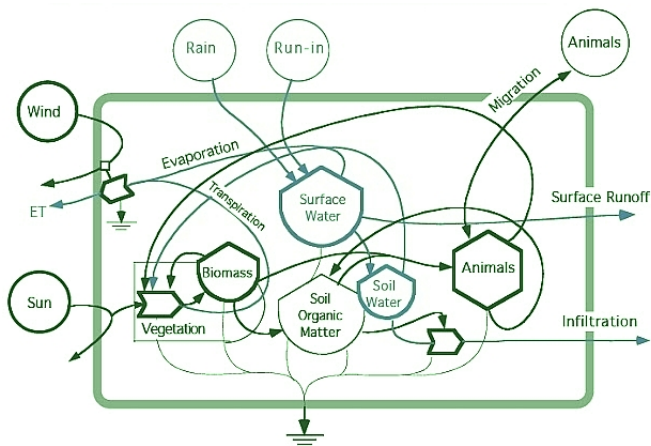
But we also need to **develop a nervous system for the planet.**

The “smart grid” is a start.

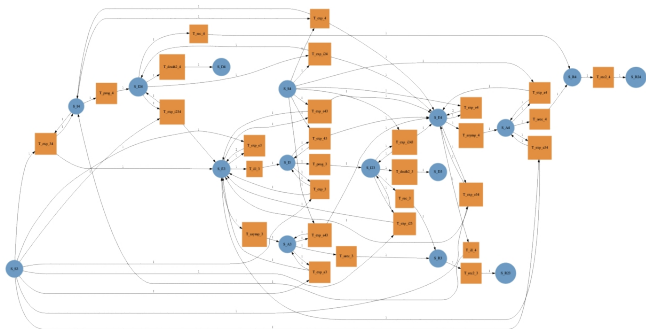


But we need a general, mathematical theory of networks.

To understand ecosystems, ultimately will be to understand networks. — B. C. Patten and M. Witkamp



The math of structured cospans led quickly to compositional models for COVID-19.



But the potential of category theory to help us design flexible networks has barely begun to be tapped.

**The 21st century will be a *very* bumpy ride.
But mathematicians can help build a saner and happier
civilization, if we put our minds to it.**

