Category theory for genetics

Rémy Tuyéras

talk by Kenny Courser

February 19, 2019

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- and more.

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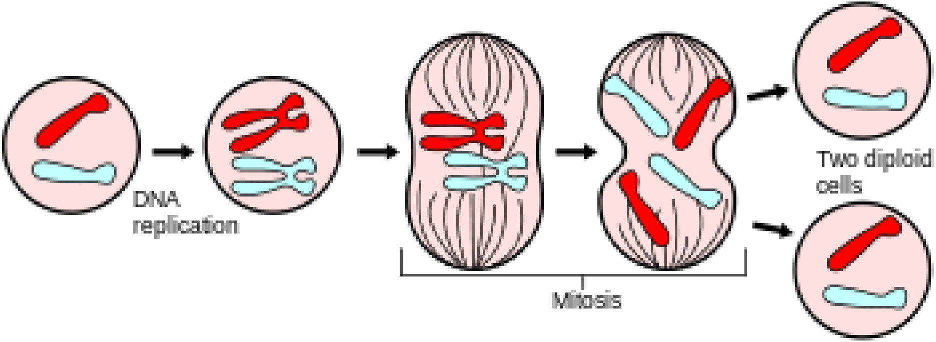

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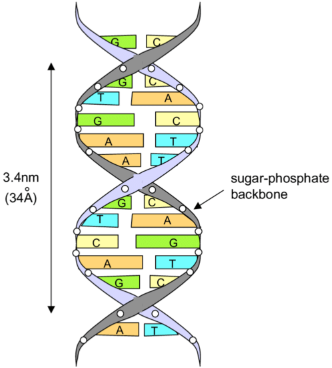
intertwined in a double helix shape.



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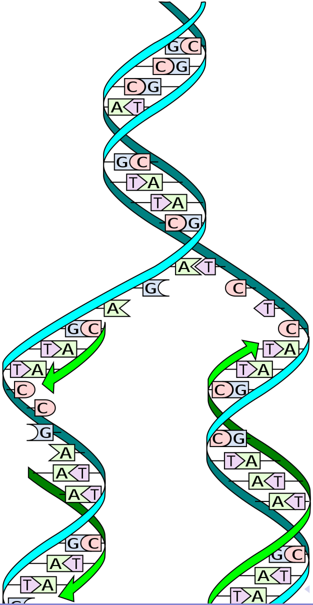

Then a ‘primer enzyme’ called *primase* binds to the unzipped strands which signals to the ‘polymerases’ where to begin replicating the DNA.

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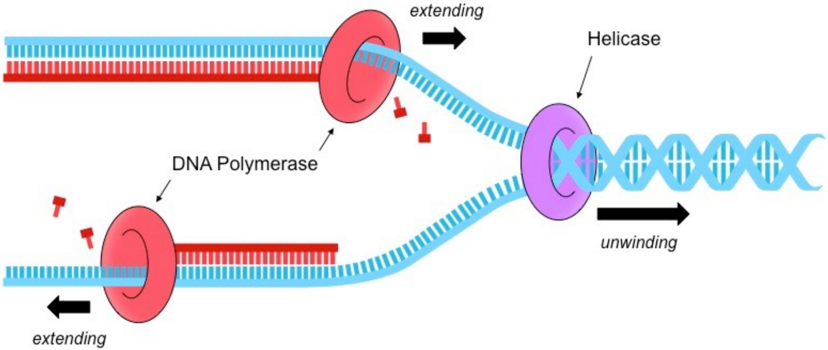
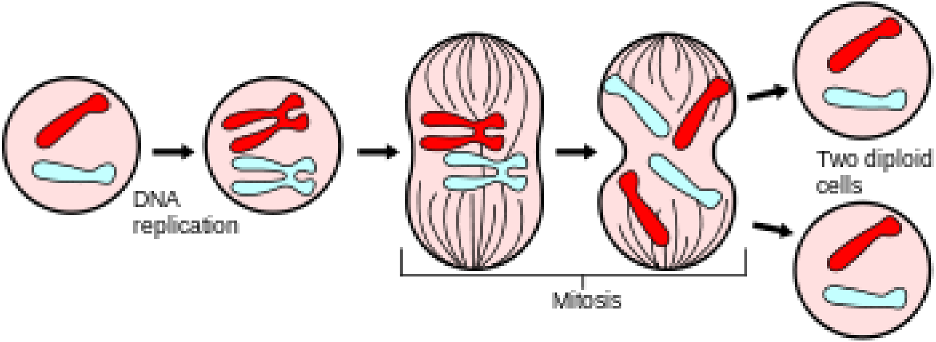


After the DNA is replicated, the cell undergoes mitosis and splits apart into two copies, each with its own copy of DNA containing one of the strands from the original cell.



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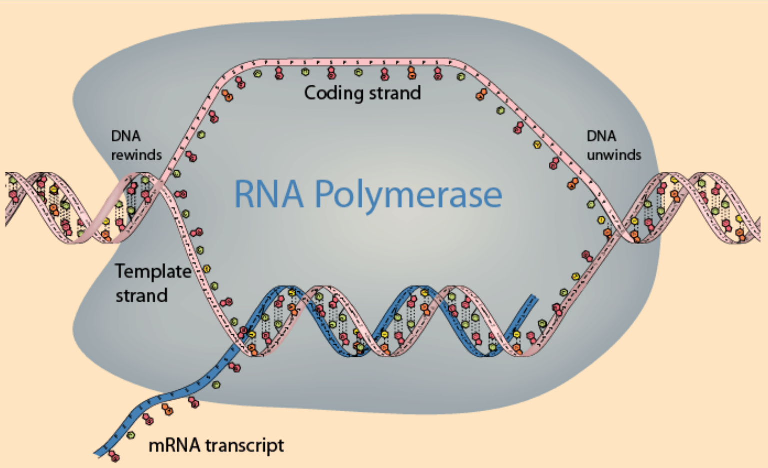
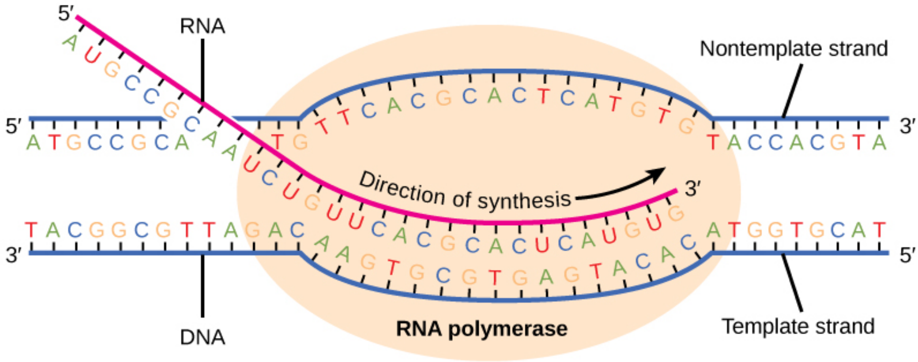
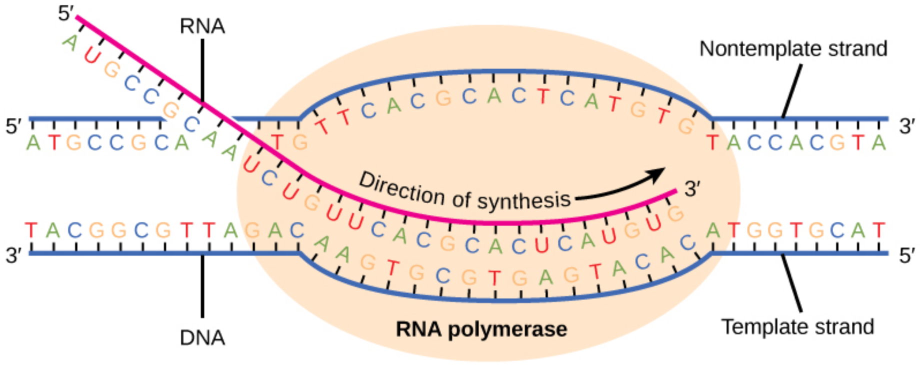



5′
RNA
Nontemplate strand
5′
3′
ATGCCGCA
TACCACGTA
Direction of synthesis
3′
TACGGCGTTAGAC
ATGGTGCAT
3′
5′
DNA
RNA polymerase
Template strand



RNA uses **U**racil instead of **T**hymine.

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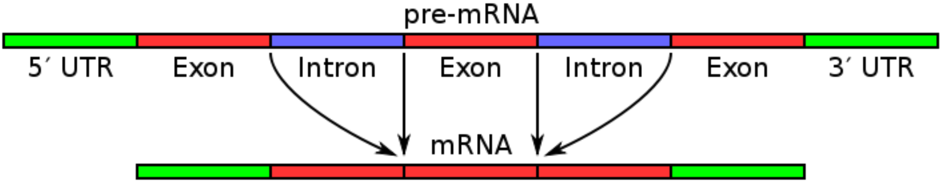

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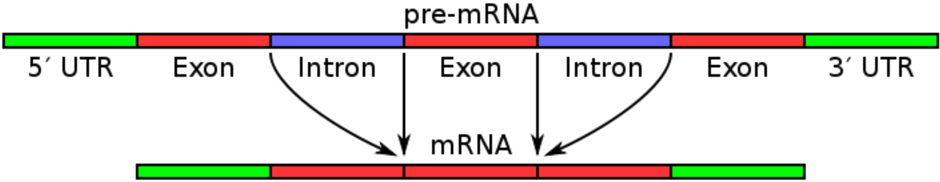

Now the mRNA strand is ready to leave the nucleus and begin translation.

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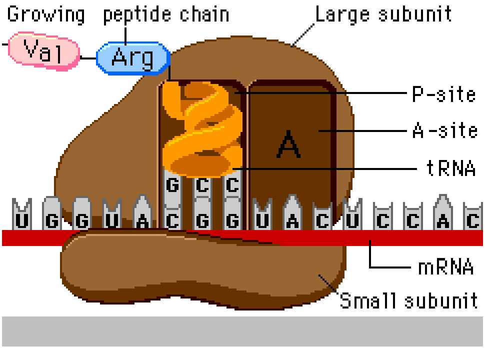
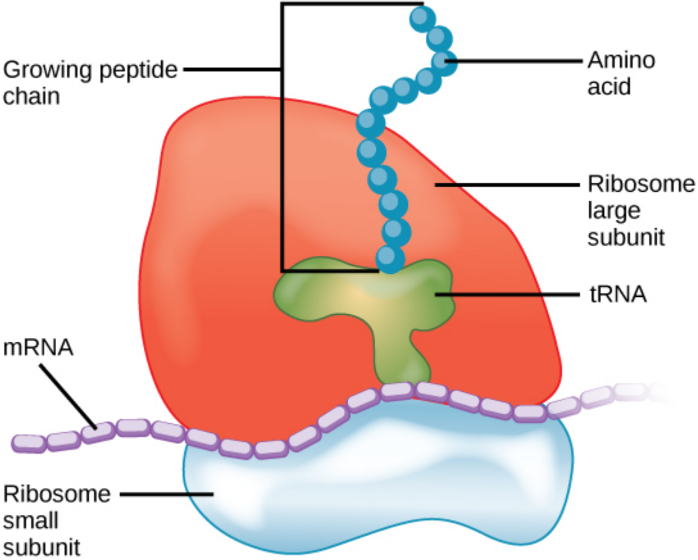
A ‘ribosomal unit’ binds to a particular ‘start codon’ (AUG) which tells the ribosomal unit where to begin translating.



Once the ribosomal unit reaches a ‘stop codon’ (UGA, UAG, UAA) the unit detaches from the mRNA strand and the sequence of amino acids (blue) created goes off to do stuff.



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The function $c\colon [n_0] \to \Omega$ then specifies how each patch is to be interpreted.

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And what about the function $c\colon [n_0] \to \Omega$?

If we take $\Omega = \{0, 1\} = \{\text{false}, \text{true}\} = \{\text{white}, \text{black}\}$ to be the Boolean preorder and define $c\colon [n_0] \to \Omega$ by

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then a visualization of the function $c \colon [3] \to \Omega$ would be:

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We will denote a **segment over** Ω simply as $(t, c)\colon [n_1] \to [n_0]$.

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{\color{green}\text{initiate}} \arrow[ur] & & {\color{red}\text{terminate}} \arrow[ul] \\
& \text{ignore} \arrow[ul] \arrow[uu] \arrow[ur] &
\end{tikzcd}
$$

With this preorder, we can consider segments (t, c) which look something like:

$$({\color{green}\bullet\bullet\bullet})({\color{blue}\bullet\bullet\bullet})(\bullet\bullet\bullet)({\color{blue}\bullet\bullet\bullet})(\bullet\bullet\bullet)({\color{blue}\bullet\bullet\bullet})({\color{red}\bullet\bullet\bullet})$$

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such that the following square commutes

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and such that $c'(f_0(i)) \leq c(i)$ for every $i \in [n_0]$.

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Proposition

Let $(\Omega, \leq)$ *be a preordered set. Then there exists a category* $\mathrm{Seg}(\Omega)$ *with:*

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- *morphisms as commutative squares*

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where $c'(f_0(i)) \leq c(i)$ *for every* $i \in [n_0]$ *and* f_1 *and* f_0 *are order preserving.*

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E.g.

$$(\circ)(\bullet\bullet\bullet\bullet)(\circ\circ)(\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet)(\bullet\bullet)(\circ\circ\circ\circ\circ)(\bullet\bullet) \quad (t, c)$$

$$\downarrow (f_1, f_0) = (id_{[n_1]}, id_{[n_0]})$$

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This says that the way one parses the patches of a segment influences the way that one parses the whole segment, e.g. codons to genes.

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\end{array}
$$

(here in this example we are supposing that $c = c' f_1$ and $f_0 = id_{[n_0]}$, meaning that the colors of the segments remain the same as do the number of patches)

We can also model ‘flexability’.

If f_1 is not an identity morphism, then the range of a segment can increase.

For example,

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(here in this example we are supposing that $c = c'f_1$ and $f_0 = id_{[n_0]}$, meaning that the colors of the segments remain the same as do the number of patches)

This allows us to insert spaces into the parsing of a segment.

Definition

Let (t, c) and (t', c') be two objects in $\mathrm{Seg}(\Omega)$. Then the two segments (t, c) and (t', c') are said to be **homologous** if their topologies t and t' are equal.

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$$(\circ)(\bullet\bullet\bullet\bullet)(\circ\circ)(\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet)(\bullet\bullet)(\circ\circ\circ\circ\circ)(\bullet\bullet) \quad (t, c)$$

and

$$(\circ)(\circ\circ\circ\circ)(\circ\circ)(\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet)(\circ\circ)(\circ\circ\circ\circ\circ)(\circ\circ) \quad (t, c')$$

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Proposition

Let $(\Omega, \leq)$ be a preorderd set and $t\colon [n_1] \twoheadrightarrow [n_0]$ an order preserving surjection. Then $\mathrm{Seg}(\Omega : t)$ is a preorder category.

Definition

If two segments (t, c) and (t', c') over Ω have the same domain $[n_1]$ then we say that the two segments (t, c) and (t', c') are **quasihomologous**.

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are quasihomologous.

Proposition

Let $(\Omega, \leq)$ be a preordered set and let n_1 be a positive integer. Then there exists a preorder category $\mathrm{Seg}(\Omega : n_1)$ whose objects are quasihomologous segments in $(\Omega, \leq)$ with domain $[n_1]$ and whose morphisms are pairs $(id_{[n_1]}, f_0)$.

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and so $Tr_1(t', c') \subset Tr_1(t, c)$.

Proposition

Let $(f_1, f_0)\colon (t, c) \to (t', c')$ *be a morphism in* $\mathrm{Seg}(\Omega)$*. If* $f_1(i) \in Tr_b(t', c')$*, then* $i \in Tr_b(t, c)$*.*

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This says that if the image of some node is truncated, then its preimage is truncated (remember, colors cannot ‘increase’).

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This says that if the image of some node is truncated, then its preimage is truncated (remember, colors cannot 'increase').

Proposition

For every $b \in \Omega$ *and nonnegative integer* n_1*, the truncation by* b *map* $(t, c) \to Tr_b(t, c)$ *extends to a functor* $Tr_b : \mathrm{Seg}(\Omega : n_1) \to \mathrm{Set}^{\mathrm{op}}$*.*

Recall there is an adjunction

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$$F(f\colon X \to Y) = f + !\colon X + \{\star\} \to Y + \{\star\}$$

The functor U is the forgetful functor which forgets the distinguished object.

Proposition

For every element $b \in \Omega$*, the map* $(t, c) \to F(Tr_b(t, c))$ *extends to a functor* $Tr_b^* \colon \mathrm{Seg}(\Omega) \to \mathrm{Set}_*^{\mathrm{op}}$ *defined as:*

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where 4 and 5 map to the distinguished element $\star$.

Now let (E, ϵ) be a pointed set and consider the following composition of functors:

$$\mathrm{Seg}(\Omega) \xrightarrow{Tr_b^*} \mathrm{Set}_*^{\mathrm{op}} \xrightarrow{\mathrm{Set}_*(\_,(E,\epsilon))} \mathrm{Set}$$

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What does E_b^ϵ do to objects?

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as well as many others.

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$$(\mathrm{TAA})(\mathrm{GATC})(\mathrm{AA}) \mapsto (\mathrm{TAA}\epsilon\epsilon)(\mathrm{GATC})(\epsilon)$$

etc.

Definition

A **cone** in a category C consists of an object $c \in \mathsf{C}$, a functor $F\colon \mathsf{A} \to \mathsf{C}$ and a natural transformation $\Delta_A(c) \Rightarrow F$ where $\Delta_A(c)$ is the constant functor mapping every object of A to $c \in \mathsf{C}$.

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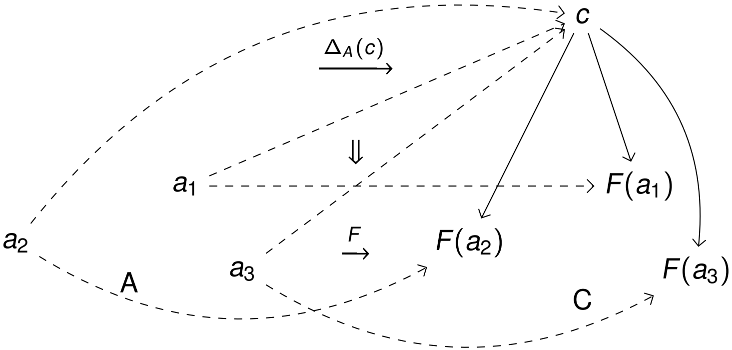

A cone over a discrete diagram A.

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$$
\begin{tikzcd}
 & & & \bullet \arrow[dlll] \arrow[dll] \arrow[dr] \arrow[drr] & \tau & & \\
\bullet & \bullet & & \cdots & \bullet & \bullet & \rho
\end{tikzcd}
$$

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\begin{tikzcd}
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\bullet & \bullet & \cdots & \bullet & \bullet & &
\end{tikzcd}
$$

If we apply the functor $Tr_b\colon \mathrm{Seg}(\Omega : n) \to \mathrm{Set}^{\mathrm{op}}$ to the cone ρ, we a get cocone in Set:

$$
\begin{tikzcd}
& & \bullet \;\; Tr_b(\tau) & & & & Tr_b(\rho) \\
\bullet \arrow[urr] & \bullet \arrow[ur] & \cdots & \bullet \arrow[ul] & \bullet \arrow[ull] & &
\end{tikzcd}
$$

From the cocone

$$Tr_b(\rho)\colon Tr_b(\theta) \Rightarrow \Delta_{\mathsf{A}} \circ Tr_b(\tau)$$

in Set, we can consider the following composite of maps through the union $\cup_{a \in \mathsf{A}} Tr_b\theta(a)$ as follows:

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$$\mathrm{colim}_{\mathsf{A}}\, Tr_b(\theta) \xrightarrow{\ e\ } \cup_{a\in\mathsf{A}} Tr_b\theta(a) \xrightarrow{\quad m\quad} Tr_b(\tau)$$

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Definition

A cone in Seg$(\Omega : n)$ is **b-distributive** if the monomorphism m above is also an epimorphism and **exactly b-distributive** if it is b-distributive and e is also a monomorphism.

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and $\mathrm{colim}_A\, Tr_1(\theta) =$

$$\{1,2,3,6,7,8,9,6',7',8',9',10,11,12,13,14,10',11',12',13',14'\}$$

and so $\mathrm{colim}_A\, Tr_1(\theta) \to Tr_1(\tau)$ is epic but not monic.

An example of an exactly distributive 1-cone in Seg(Ω : 18) over the discrete diagram $A = \{\bullet\bullet\bullet\}$ is given by:

$$(\bullet\bullet\bullet)(\circ\circ)(\bullet\bullet\bullet\bullet)(\bullet\bullet\bullet\bullet\bullet)(\circ\circ\circ)(\circ) \leq (\circ\circ\circ)(\circ\circ)(\bullet\bullet\bullet\bullet)(\circ\circ\circ\circ\circ)(\circ\circ\circ)(\circ)$$

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$$\begin{tikzcd}
& \tau \quad (\bullet\bullet\bullet)(\circ\circ)(\bullet\bullet\bullet\bullet)(\bullet\bullet\bullet\bullet\bullet)(\circ\circ\circ)(\circ) \arrow[dl] \arrow[d] \arrow[dr] & \\
(\circ\circ\circ)(\circ\circ)(\bullet\bullet\bullet\bullet)(\circ\circ\circ\circ\circ)(\circ\circ\circ)(\circ) & {} \arrow[d] & (\circ\circ\circ)(\circ\circ)(\circ\circ\circ\circ)(\bullet\bullet\bullet\bullet\bullet)(\circ\circ\circ)(\circ) \\
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\end{tikzcd}$$

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Here's an example of an exactly distributive 1-cone in which common black patches are identified via the diagram A:

$$
\begin{tikzcd}
\tau & & (\bullet\,\bullet\,\bullet)(\bullet\bullet)(\bullet\,\bullet\,\bullet)(\bullet\,\bullet\,\bullet\bullet) \arrow[dll] \arrow[dd] \arrow[drr] & & \\
(\circ\,\circ\,\circ)(\bullet\bullet)(\bullet\,\bullet\,\bullet)(\circ\,\circ\,\circ\circ) \arrow[ddrr, bend right] & & & & (\circ\,\circ\,\circ)(\bullet\bullet)(\circ\,\circ\,\circ)(\bullet\,\bullet\,\bullet\bullet) \arrow[ddll, bend left] \\
& & (\bullet\,\bullet\,\bullet)(\bullet\bullet)(\circ\,\circ\,\circ)(\circ\,\circ\,\circ\circ) \arrow[d] & & \\
& & (\circ\,\circ\,\circ)(\bullet\bullet)(\circ\,\circ\,\circ)(\circ\,\circ\,\circ\circ) & &
\end{tikzcd}
$$

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- Duplication
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Consider the following pair of morphisms in Seg(Ω):

$$f_1 : (\bullet_1 \bullet_2 \bullet_3) \to (\bullet_1 \bullet_2 \bullet_3)(\circ \circ \circ)$$

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The functor E_1^ϵ applied to either f_1 or f_2 is an identity which sends any word of length 3 in (E, ϵ) to itself, e.g.

$$(\mathrm{A\ T\ G}) \xrightarrow{E_1^\epsilon(f_1)} (\mathrm{A\ T\ G})$$

Consider the following exactly 1-distributive cone:

$$
\begin{tikzcd}[column sep=small]
& (\bullet_1\ \bullet_2\ \bullet_3)(\bullet_4\ \bullet_5\ \bullet_6) \arrow[dl] \arrow[dr] & \\
(\bullet_1\ \bullet_2\ \bullet_3)(\circ_4\ \circ_5\ \circ_6) & & (\circ_1\ \circ_2\ \circ_3)(\bullet_4\ \bullet_5\ \bullet_6)
\end{tikzcd}
$$

Consider the following exactly 1-distributive cone:

$$
\begin{array}{ccc}
& (\bullet_1\ \bullet_2\ \bullet_3)(\bullet_4\ \bullet_5\ \bullet_6) & \\
\swarrow & & \searrow \\
(\bullet_1\ \bullet_2\ \bullet_3)(\circ_4\ \circ_5\ \circ_6) & & (\circ_1\ \circ_2\ \circ_3)(\bullet_4\ \bullet_5\ \bullet_6)
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$$\mu\colon E_1^{\epsilon}((\bullet_1\bullet_2\bullet_3)(\bullet_4\bullet_5\bullet_6)) \to E_1^{\epsilon}((\bullet_1\bullet_2\bullet_3)(\circ_4\circ_5\circ_6))\times E_1^{\epsilon}((\circ_1\circ_2\circ_3)(\bullet_4\bullet_5\bullet_6))$$

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This function maps any pair of words of length 3 to their concatenation, e.g.:

$$(A\ T\ G, G\ A\ T) \to A\ T\ G\ G\ A\ T$$

If we precompose the map μ^{-1}:

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CRISPR

CRISPR - Clustered Regularly Interspaced Short Palindromic Repeats

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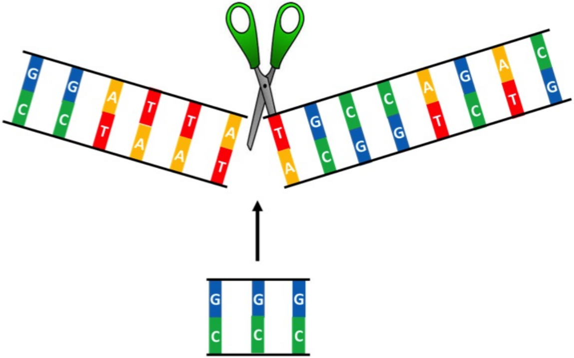

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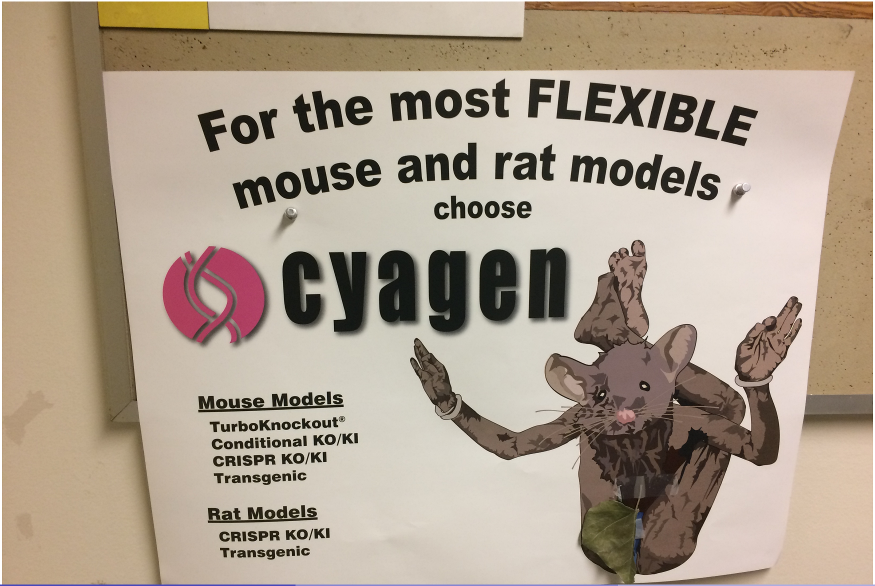

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and we wish to rewrite the portion C G T as T T C.

$$\mathrm{A\ T\ C\ G\ T\ C} \mapsto \mathrm{A\ T\ T\ T\ C\ C}$$

In order to do this, we need to first select the subsegment C G T inside of the segment A T C G T C and then replace it with T T C.

The word A T C G T C is an element of the set

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If we let f denote the map

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$$E_1^{\epsilon}((\bullet\bullet)(\bullet\bullet\bullet)(\bullet))\times E_1^{\epsilon}((\circ\circ)(\bullet\bullet\bullet)(\circ))\xrightarrow{(E_1^{\epsilon}(f),E_1^{\epsilon}(id))}E_1^{\epsilon}((\bullet\bullet)(\circ\circ\circ)(\bullet))\times E_1^{\epsilon}((\circ\circ)(\bullet\bullet\bullet)(\circ))$$

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$$\mu\colon E_1^{\epsilon}((\bullet\bullet)(\bullet\bullet\bullet)(\bullet)) \to E_1^{\epsilon}((\bullet\bullet)(\circ\circ\circ)(\bullet)) \times E_1^{\epsilon}((\circ\circ)(\bullet\bullet\bullet)(\circ))$$

The following cone is exactly 1-distributive:

$$
\begin{tikzcd}
 & (\bullet\bullet)(\bullet\bullet\bullet)(\bullet) \arrow[dl] \arrow[dr] & \\
(\bullet\bullet)(\circ\circ\circ)(\bullet) & & (\circ\circ)(\bullet\bullet\bullet)(\circ)
\end{tikzcd}
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The inverse is given by:

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The image of the pair (A T C G T C, T T C) is A T T T C C.

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Let $(\Omega, \leq)$ be the Boolean preorder, $b = 1 =$ true, $(A, \epsilon) =$ {A,C,G,T,ϵ} and $(B, \epsilon) =$ {A,C,G,U,ϵ} and define a bijection of pointed sets $f\colon (A, \epsilon) \to (B, \epsilon)$ by

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$$A \mapsto U$$

$$T \mapsto A$$

$$G \mapsto C$$

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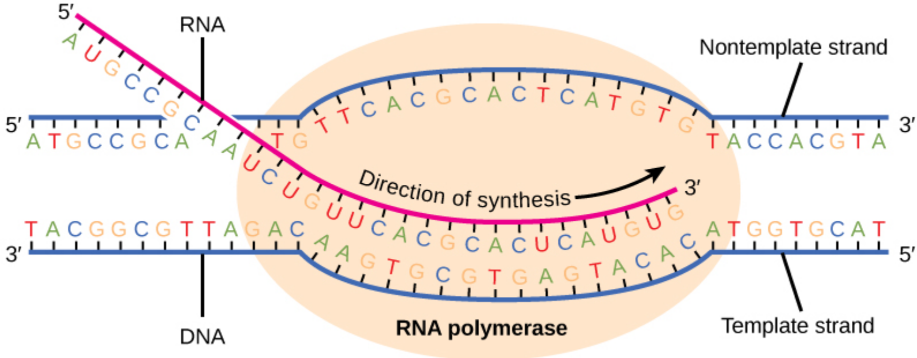

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These projection maps p and q induce natural transformations

$$p_b^* \colon (E \times E, (\epsilon, \epsilon)) \Rightarrow (E, \epsilon)$$

and

$$q_b^* \colon (E \times E, (\epsilon, \epsilon)) \Rightarrow (E, \epsilon)$$

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Inversions

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E.g. for $rv_5 \colon [5] \to [5]$, we have

$$1 \mapsto 5$$
$$2 \mapsto 4$$
$$3 \mapsto 3$$
$$4 \mapsto 2$$
$$5 \mapsto 1$$

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The map $(t, c) \mapsto (t, c)^\dagger$ induces an endofunctor $Inv\colon \mathrm{Seg}(\Omega) \to \mathrm{Seg}(\Omega)$.

If we take $E = \{\text{A,C,G,T}, \epsilon\}$ as our pointed set and $b \in \Omega = \{0 \leq 1\}$, then the functor

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$$E_b^\epsilon \Rightarrow E_b^\epsilon \circ Inv$$

$$E_b^\epsilon(t,c) \to E_b^\epsilon \circ Inv(t,c) = E_b^\epsilon(t,c)^\dagger$$

which maps any word to its inversion.

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$$\mathrm{AGTAGC} \mapsto \mathrm{CGATGA}$$

$$\mathrm{CTTACA} \mapsto \mathrm{ACATTC}$$

Inversion is useful for interpreting the ‘lagging strand’ (red) having to be read backwards.