THE LANDAUER LIMIT AND THERMODYNAMICS OF BIOLOGICAL SYSTEMS

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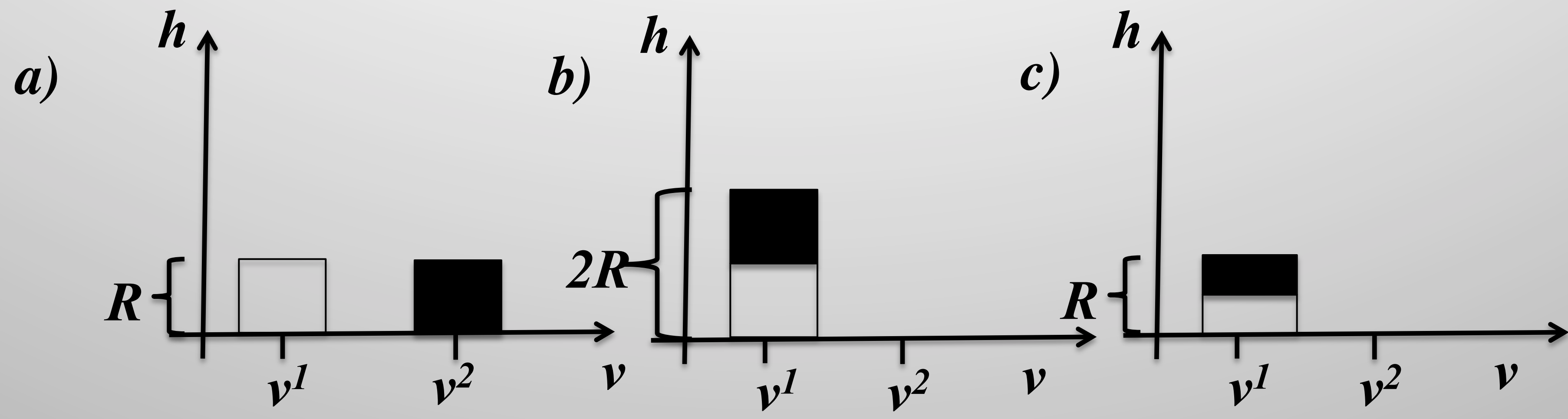
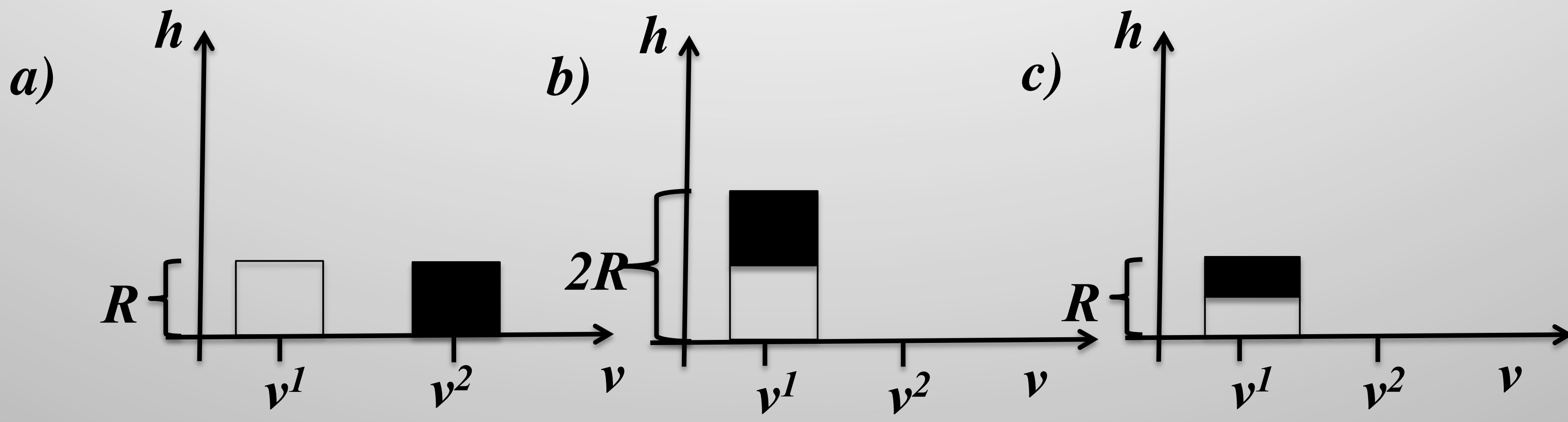
HEAT OF ERASING A BIT



***Thermodynamic cost to erase a bit - the minimal amount of entropy that must be expelled to the environment - is* ln[2]**

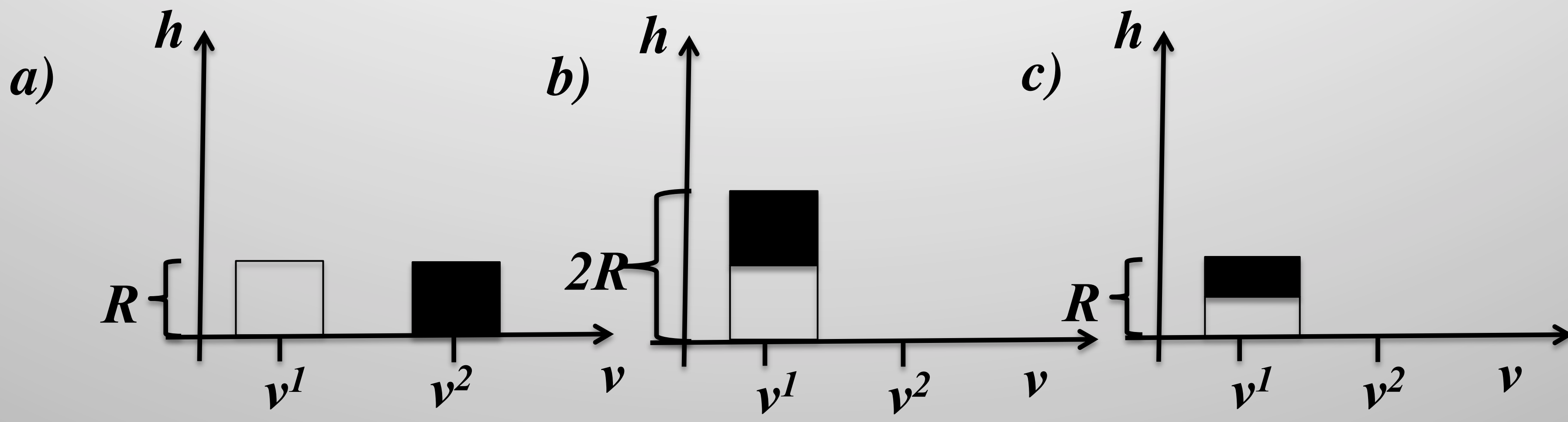
HEAT OF ERASING A BIT



- **Crucially, *DO* know precise pre-erasure value of bit**
  - ***After all, a computer is useless if don't know its initial state***

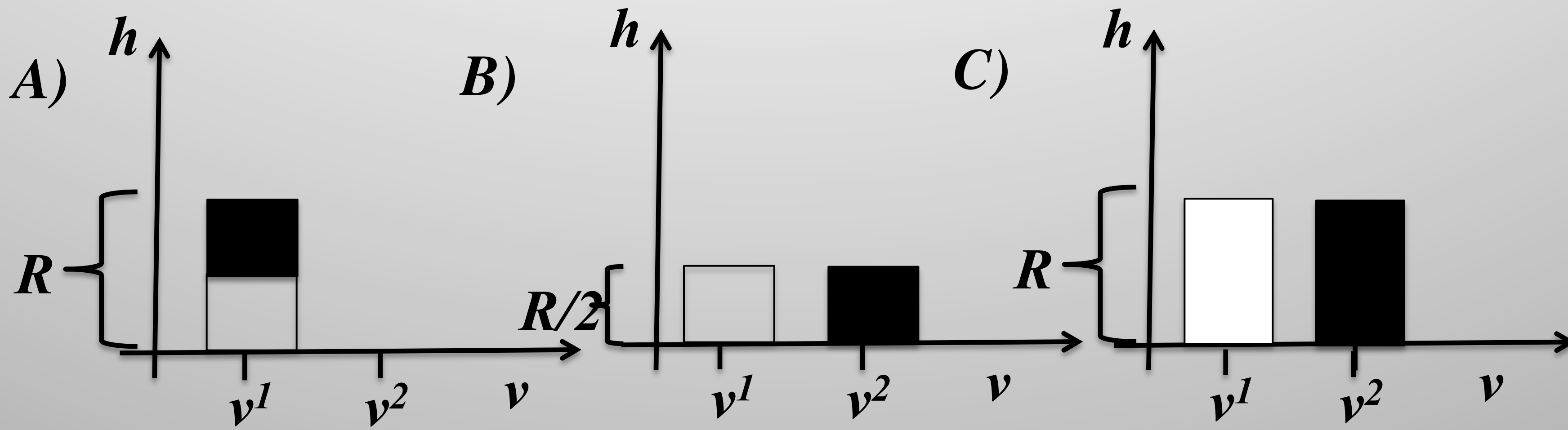
Bennett, 2003*: "If erasure is applied to random data, the operation may be thermodynamically reversible … but if it is applied to known data, it is thermodynamically irreversible."*

HEAT OF ERASING A BIT



- **Crucially, *DO* know precise pre-erasure value of bit**
  - ***After all, a computer is useless if don't know its initial state***
- **In fact, the prior distribution over the pre-erasure value – and in particular the entropy of that prior – is irrelevant**
- **Requires careful engineering to make this property hold**
- **"Local detailed balance" does *not* hold**

REFRIGERATION BY RANDOMIZING A BIT



- **Example:** ***Adiabatic demagnetization***
- **Exploited in modern engineering:**
  - ***Noisy error correction computing***
  - **Real (not "pseudo") random number generators**

SOME ERASING AND SOME RANDOMIZATION

- ***What is the thermodynamic cost for an arbitrary conditional distribution from $X = \{0, 1, 2, 3\}$ into itself?***

- ***E.g., what if***
  - ***0 and 1 go to 0 (as in bit erasure);***

    i.e., $P(0 \mid 0) = P(0 \mid 1) = 1$
  - ***2 goes to 0 with probability .8, stays the same otherwise;***

    i.e., $P(0 \mid 2) = .8, P(2 \mid 2) = .2$
  - ***3 goes to 2 with probability .4, and to 0 with probability .6;***

    i.e., $P(2 \mid 3) = .5, P(0 \mid 3) = .6$

THERMODYNAMIC COST

$$\mathbb{E}(cost) = \sum_{v_t, v_{t+1}} \pi(v_{t+1} \mid v_t) P(v_t) \ln \left[\sum_{v'_t} \pi(v_{t+1} \mid v_t) \right]$$
$$= I(W_{t+1}; V_{t+1}) - I(W_t; V_t)$$

- ***where v_t is the observable v's value at time t;***
- ***π(. | .) is the conditional distribution of dynamics;***
- ***I(. ; .) is mutual information;***
- ***W is unobserved degrees of freedom;***

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Example:

In a 2-to-1 map, π(0 | 0) = π(0 | 1) = 1,
so expected cost equals* ln*[2]

BOUNDS ON THERMODYNAMIC COST

Given the evolution kernel $\pi(.\,|\,.)$, as one varies $P(v_t)$:

$$0 \;\leq\; \mathbb{E}(cost) + H(V_t) + H(V_{t+1}) \;\leq\; \log[|V|] - \max_{v_t}\left[KL\big(\pi(V_{t+1} \mid a) \;||\; \Pi^{+}(V_{t+1})\big)\right]$$

- ***where v_t is the observable v's value at time t;***
- **H(.)** ***is Shannon entropy;***
- **KL(. || .)** ***is KL divergence;***
- $\Pi^{+}(v_{t+1}) \;=\; \sum_{v_t} \pi(v_{t+1} \,|\, v_t) \,/\, |V|$

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Example: In a 2-to-1 map both bounds are tight:

Thermodynamic cost is the drop in Shannon entropies over V

K'TH ORDER MARKOV CHAINS

For a k'th order Markov chain, thermodynamic cost during a single step is bounded below by

$$L \equiv H(V_t \mid V_{t+1}, \ldots, V_{t+k-1}) - H(V_{t+k} \mid V_{t+1}, \ldots, V_{t+k-1})$$

and above by

Very messy expression

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If P(v) has reached stationarity, lower bound is

$$\mathbb{E}(cost) \;=\; I(V_t; V_{t+1}, \ldots, V_{t+k-1}) - I(V_{t+k}; V_{t+1}, \ldots, V_{t+k-1})$$

(Cf. Still et al. 2012)

THERMODYNAMIC COST

Tons of other fun results, including:

1) Second law: Thermodynamic cost is non-negative for any process that goes from distribution A to distribution B and then back to distribution B

2) Examples of coarse-graining (in the statistical physics sense) that increase thermodynamic cost

3) Examples of coarse-graining that decrease thermo. cost

4) Implications for optimal compiler design

5) Analysis of thermodynamic cost of Hidden Markov Models

IMPLICATIONS FOR DESIGN OF BRAINS

- ***$P(x_t)$ a dynamic process outside of a brain;***
- ***Natural selection favors brains that:***
  - ***(generate v_t's that) predict future of x accurately;***

    but …

  - ***not generate heat that needs to be dissipated;***
  - ***not require free energy from environment (need to create all that heat)***

Natural selection favors brains that:

1) Accurately predict future (*quantified with a fitness function*);
2) Using a prediction program with minimal thermo. cost

IMPLICATIONS FOR BIOCHEMISTRY

- ***Natural selection favors (phenotypes of) a prokaryote that:***
  - ***(generate v_t's that) maximize fitness;***

    but …

  - ***not generate heat that needs to be dissipated;***
  - ***not require free energy from environment (need to create all that heat)***

Natural selection favors prokaryotes that:

1) Behave as well as possible (*quantified with a fitness function*);
2) While implementing behavior with minimal thermo. cost

COMPLEXITY DYNAMICS OF BIOSPHERE

$$\mathbb{E}(cost) = \sum_{v_t, v_{t+1}} \pi(v_{t+1} \mid v_t) P(v_t) \ln \left[\sum_{v'_t} \pi(v_{t+1} \mid v_t) \right]$$

- ***where v_t is the observable v's value at time t;***
- ***π(. | .) is the conditional distribution of dynamics;***

N.b., thermodynamic cost varies with t:

For what kernels π(. | .) does thermo. cost increase with time?

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- ***where v_t is the observable v's value at time t;***
- ***π(. | .) is the conditional distribution of dynamics;***

Plug in π(. | .) of terrestrial biosphere:

Does thermo. cost of biosphere behavior increase with time?

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Plug in π(. | .) of terrestrial biosphere:

How far is thermo. cost of biosphere from upper bound of solar free energy flux?