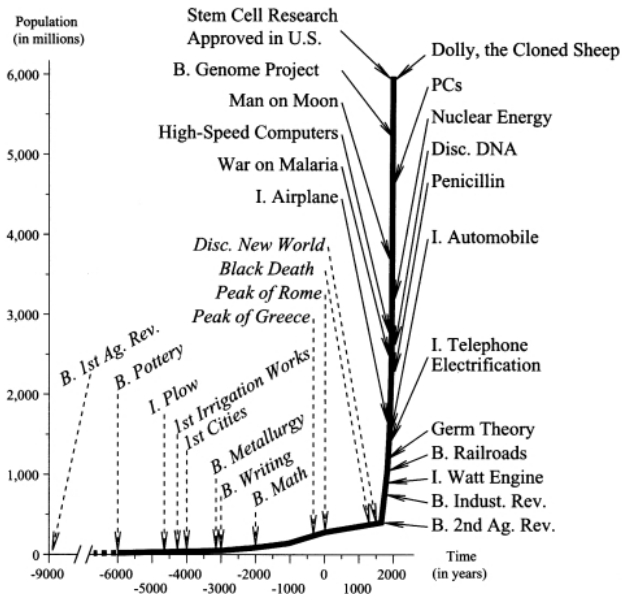


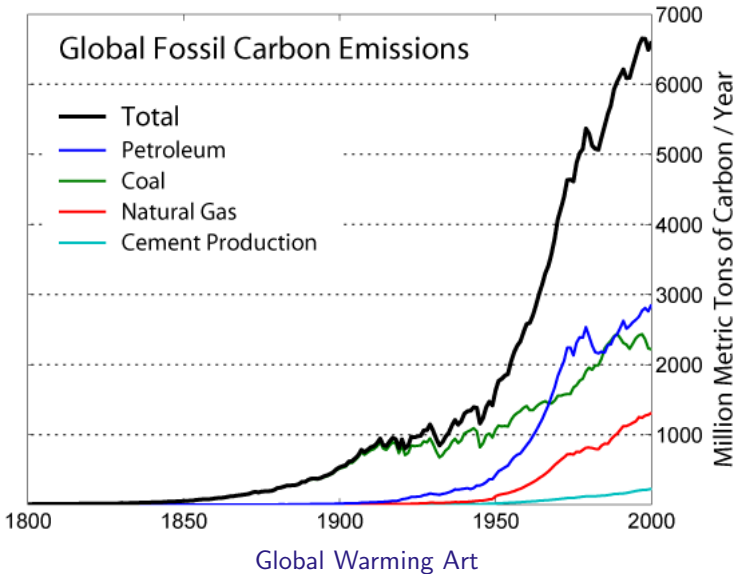
# THE MATHEMATICS OF PLANET EARTH

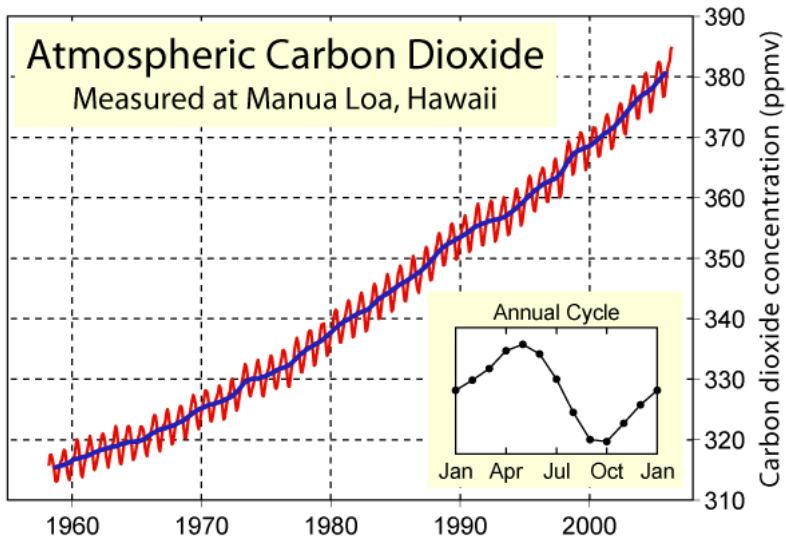


**John Baez**  
**October 2, 2012**



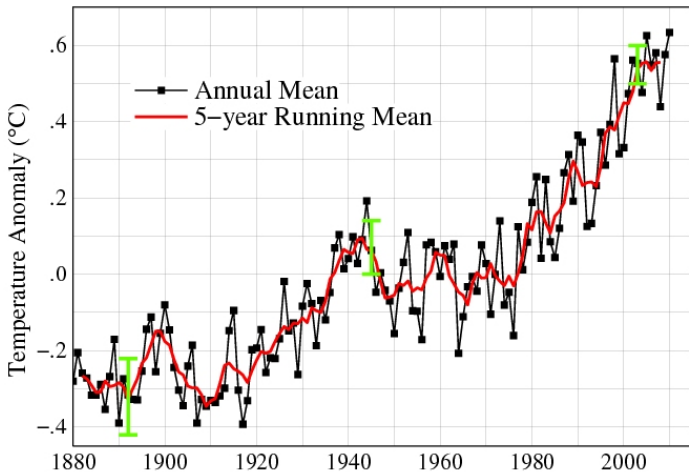
Robert Fogel - *The Escape from Hunger and Premature Death, 1700-2100*





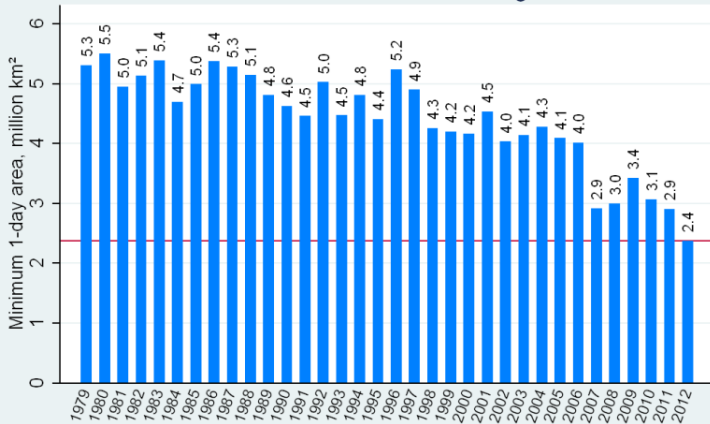
The Keeling Experiment — Global Warming Art

## Global Land–Ocean Temperature Index



NASA Goddard Institute of Space Science

## Minimum CT Arctic sea ice area through 9/2/2012

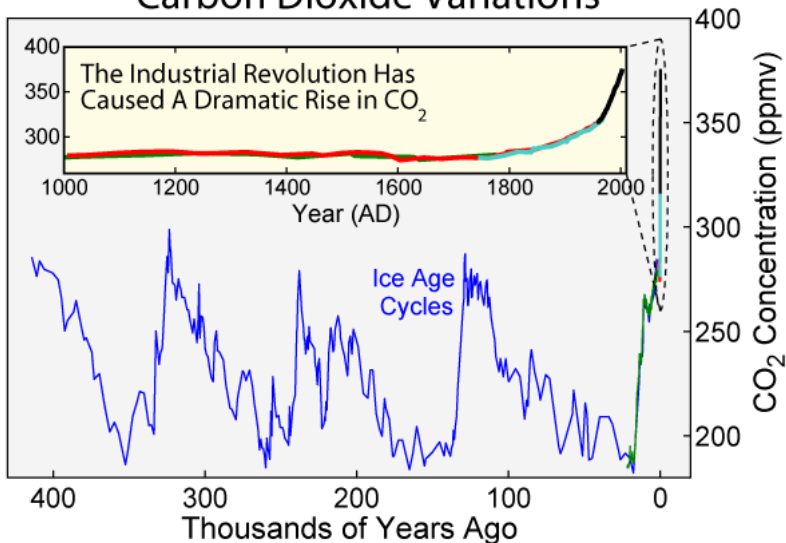


graph: L Hamilton

data: Cryosphere Today

## The Cryosphere Today

# Carbon Dioxide Variations



Antarctic ice cores and other data — Global Warming Art

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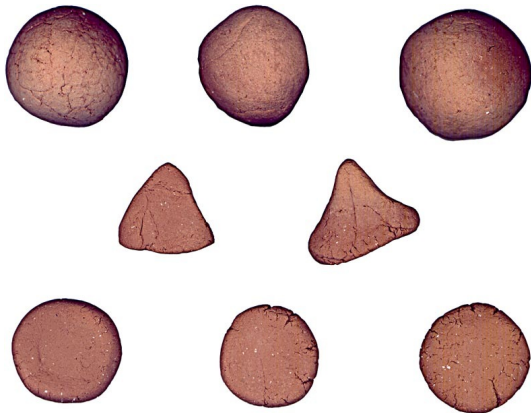
By now we use about 25% of all plant biomass grown worldwide! If this reaches 100% there will be, in some sense, no 'nature' separate from humanity.

Starting shortly after the end of the last ice age, the agricultural revolution led to:

- ▶ surplus grain production, and thus kingdoms and slavery.
- ▶ *astronomical mathematics* for social control and crop planning.
- ▶ *geometry* for measuring fields and storage containers.
- ▶ *written numbers* for commerce.

Consider the last...

Starting around 8,000 BC, in the Near East, people started using 'tokens' for contracts: little geometric clay figures that represented things like sheep, jars of oil, and amounts of grain.



MS 5067/1-8  
Neolithic plain counting tokens. Near East, ca. 8000–3500 BC

The Schøyen Collection

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Eventually they gave up on the tokens. The marks on tablets then developed into the Babylonian number system! The transformation was complete by 3,000 BC.

1		11		21		31		41		51	
2		12		22		32		42		52	
3		13		23		33		43		53	
4		14		24		34		44		54	
5		15		25		35		45		55	
6		16		26		36		46		56	
7		17		27		37		47		57	
8		18		28		38		48		58	
9		19		29		39		49		59	
10		20		30		40		50			

J. J. O'Connor and E. F. Robertson, Babylonian Numerals

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By 1700 BC the Babylonians could compute  $\sqrt{2}$  to 6 decimals:

$$1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} \approx 1.414213\dots$$



Yale Babylonian Collection, YBC7289

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Math may undergo a transformation just as big as it did in the Agricultural Revolution.

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Even better, these machines should spread without human intervention.





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For sophisticated ecotechnology we need to pay attention to what's already known—[permaculture](#), [systems ecology](#) and so on. But better mathematics could help.

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But CO<sub>2</sub> also helps plants grow leaves. [Bounoua et al](#) say this effect would cool the land by 0.6 °C with doubled CO<sub>2</sub>.

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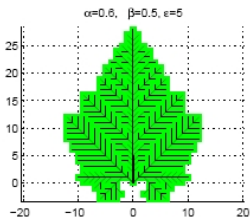
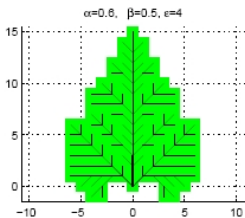
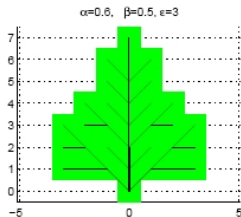
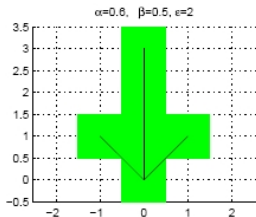
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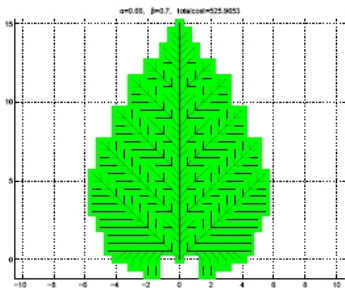
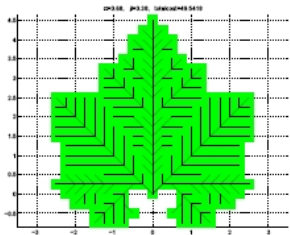


Yes! A mathematician at U.C. Davis, Qinglan Xia, has written a paper called *The Formation of a Tree Leaf*.

He models a leaf as a union of square cells centered on a grid, together with 'veins' forming a weighted directed graph from the centers of the cells to the root. The leaf grows new cells at the boundary while minimizing a certain cost function.



The cost function depends on two parameters. Changing these gives different leaf shapes:



## Qinglan Xia's work is definitely math:

**Lemma 3.8.** *Suppose  $(\Omega, G)$  is an  $(\epsilon, h)$  leaf and  $(\mu, \Theta) = \phi_h(\Omega, G)$ . Then the total mass of the Radon measure is bounded above by*

$$M(\mu) \leq \pi(R_\epsilon + h)^2$$

*and the total variation of the vector measure  $\Theta$  is bounded by*

$$M(\Theta) \leq \epsilon \pi^{2-\alpha} (R_\epsilon + h)^{4-2\alpha}.$$

*Proof.* Since  $\Omega \subset B_{R_\epsilon}(0)$ , the mass of  $\mu$  is given by

$$\begin{aligned} M(\mu) &= \|\Omega\| h^2 \\ &= \text{area} \left( \bigcup_{x \in \Omega} \left\{ x + \left[ -\frac{h}{2}, \frac{h}{2} \right] \times \left[ -\frac{h}{2}, \frac{h}{2} \right] \right\} \right) \\ &\leq \text{area}(B_{R_\epsilon+h}(0)) = \pi(R_\epsilon + h)^2. \end{aligned}$$

Also, since  $w(e) \leq \|\Omega\| h^2$  for each  $e \in E(G)$ , the total variation of  $\Theta$  is given by

$$\begin{aligned} M(\Theta) &= \sum_{e \in E(G)} w(e) \text{length}(e) \\ &\leq (\|\Omega\| h^2)^{1-\alpha} \sum m_\beta(e^+) (w(e))^\alpha \text{length}(e) \end{aligned}$$

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It's just beginning to be born. At the [Azimuth Project](#) we're trying to help it along.