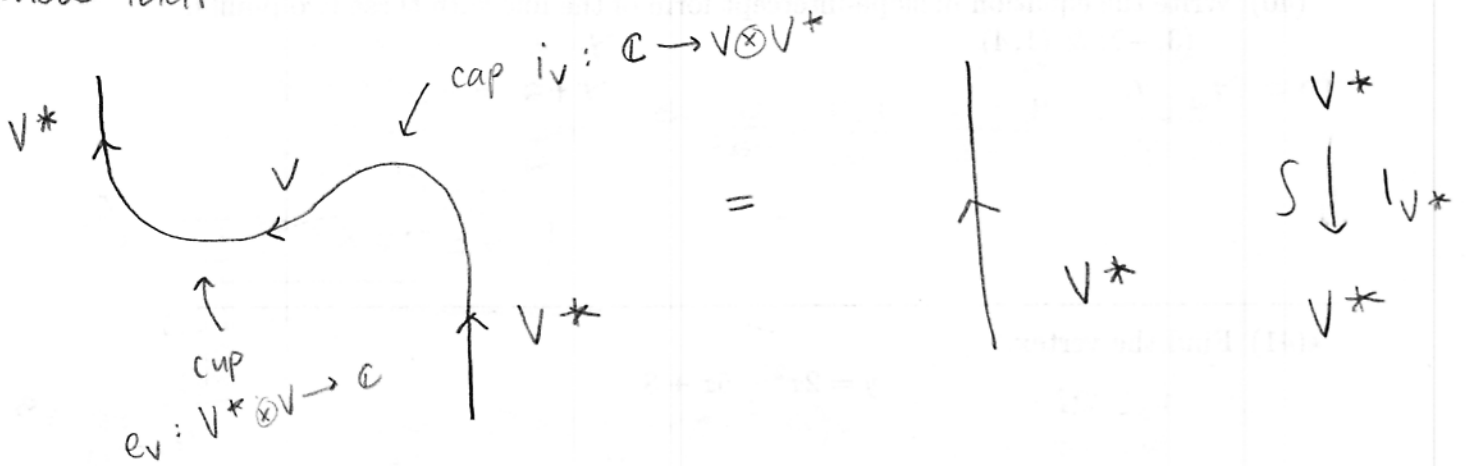


Show that



$$f \in V^* \xrightarrow{\quad} V^*$$

$$\downarrow \quad \quad \downarrow \text{isomorphic}$$

$$f \otimes 1 \in V^* \otimes \mathbb{C} \xrightarrow{\quad} V^* \otimes \mathbb{C}$$

$$\downarrow \text{under } i_V \quad \quad \downarrow i_{V^*} \otimes i_V$$

$$f \otimes (\sum_i e_i \otimes e_i) \in (V^* \otimes (V) \otimes V^*)$$

tensor products are  
regroup ( )<sup>assoc.</sup> ✓

$$\begin{matrix} \text{under } e_V \} \downarrow & \downarrow \text{under } i_{V^*} & \downarrow & e_V \otimes i_{V^*} \\ \sum_i f(e_i) \otimes e_i \in & \mathbb{C} \otimes V^* & & \end{matrix}$$

$$\downarrow \quad \quad \downarrow \text{isomorphic}$$

$$\sum_i f(e_i) e_i \in V^*$$

10/16/00

Now we need to show

$$f = \sum_i f(e_i)e_i$$

Recall:  $v = \sum v_i e_i$

$$f(v) = f\left(\sum_i v_i e_i\right) = \sum_i f(e_i)v_i \quad \text{since } f \in V^*$$

↙ equal.

$$\sum_i f(e_i)e_i(v) = \sum_i f(e_i)v_i$$

└──  
i<sup>th</sup> component  
of  $v$  in the  
basis  $\{e_i\}$

right!  
good!

we can pull  $f$  in  
since it's linear!