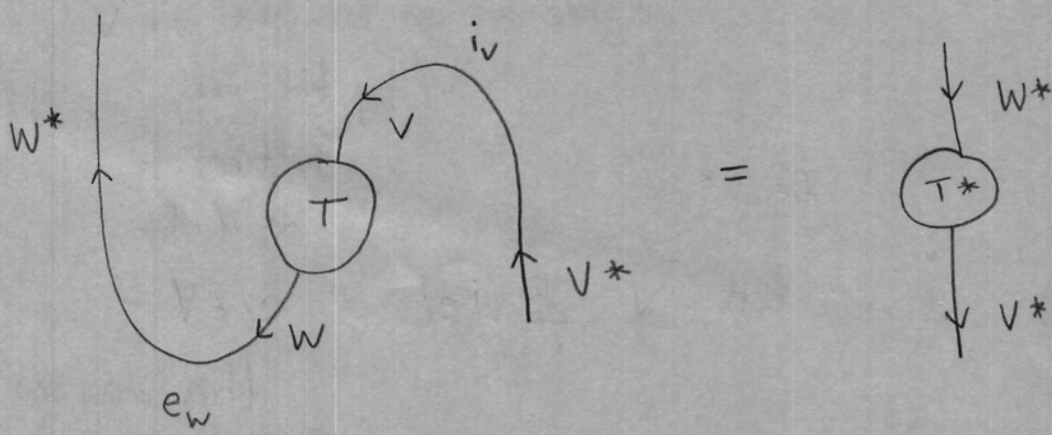
Alissa Crans

HW # 3

10/23/00

Given $T: V \longrightarrow W$

Show the following is the same:



$$
\begin{array}{rc}
 & W^* \\
\wr & \downarrow \\
 & W^* \otimes \mathbb{C} \\
1_{W^*} \otimes i_V & \downarrow \\
 & W^* \otimes (V \otimes V^*) \\
1_{W^*} \otimes T \otimes 1_{V^*} & \downarrow \\
 & (W^* \otimes W) \otimes V^* \\
e_W \otimes 1_{V^*} & \downarrow \\
 & \mathbb{C} \otimes V^* \\
\wr & \downarrow \\
 & V^*
\end{array}
$$

$$
\begin{array}{l}
f \\
\quad \downarrow \\
f \otimes 1 \\
\quad \downarrow \\
f \otimes 1_V = f \otimes \sum_i e_i \otimes e^i \\
\quad \downarrow \\
f \otimes \sum_i \underbrace{T(e_i) \otimes e^i}_{\in\, W \otimes V^*} \\
\quad \downarrow \\
\sum_i \underbrace{f(T(e_i))}_{\in\, \mathbb{C}} \otimes e^i \\
\quad \downarrow \\
\sum_i f(T(e_i)) e^i
\end{array}
$$

$\longrightarrow$ over

Recall: $T : V \longrightarrow W$

$f \in W^*$ so $f : W \longrightarrow \mathbb{C}$

$T^* : W^* \longrightarrow V^*$

Now show

$$T^*(f) = \sum_i f(T(e_i))e^i$$

We must check that they agree on an element $v \in V$.

(by the way, suffices to do it for $e_i \in V$ — this may or may not make your job easier)

$f \in W^*$

$$T^*(f)(v) = f(T(v))$$

But $v = \sum_i v^i e_i$

$$= f\left(T\left(\sum_i v^i e_i\right)\right)$$

T is linear

$$= f\left(\sum_i v^i T(e_i)\right)$$

f is linear

$$= \sum_i f(T(e_i))\, v^i$$

same

$$\sum_i f(T(e_i))\, e^i(v) = \sum_i f(T(e_i))\, v^i$$