Witten (1989) – described how the symmetric monoidal category \( \text{Rep}(\text{SU}(2)) \) can be "deformed" (as a function of \( q = e^{i\theta} \in U(1) \)) into a braided but nonsymmetric monoidal category "\( \text{Rep}(\text{SU}_q(2)) \)" where \( \text{SU}_q(2) \) is a quantum group (not a group, but a thing whose representations form a braided monoidal category), already discovered by Fadeev, Reshetikhin, & the Russian school (working on completely integrable systems). The new thing about Witten's approach is that it got a hold of the braided monoidal category directly (i.e. without knowing about \( \text{SU}_q(2) \)), using CFT.

Turzov & Viro (1992) – showed how the Ponzano-Regge model, defined using \( \text{Rep}(\text{SU}(2)) \) could be improved by using \( \text{Rep}(\text{SU}_q(2)) \) instead! (Actually, they only learned of the Ponzano-Regge model and its relation to 3d quantum gravity later.)

In the P-R model we can calculate an amplitude for any triangulated compact 3-manifold with edge lengths given by spins \( j = 0, \frac{1}{2}, \ldots \). Also, the sum of these...
of these amplitudes over all labellings diverges. But $SU_q(2)$ has only finitely many irreps: $j = 0, \frac{1}{2}, \ldots, \frac{1}{2}$ where

$$ q = e^{\frac{2\pi i}{k+2}}. $$

So: in Turaev & Viro's version of the P-R model, the sum over labellings is a finite sum! Even better, the result doesn't depend on the choice of triangulation! Even better, we get a TQFT

$$ Z : 3\text{Cob} \to \text{Hilb} $$

which gives these amplitudes for compact 3-manifolds.

Barrett & Westbury (1992) — showed that Turaev & Viro's recipe for getting a 3d TQFT from $\text{Rep}(SU_q(2))$ generalized to a large class of monoidal categories — called "spherical categories." (In fact they did this before reading Turaev & Viro's work, & unlike T-V, they knew the relation to 3d QG.)

Fukuma, Hosono, Kawai (1992) — used a similar trick to get 2d TQFTs:

$$ Z : 2\text{Cob} \to \text{Hilb} $$

from certain nice monoids (in fact semisimple algebras)
Louis Crane (1993) — Published a paper entitled "Categorical Physics" in which he explicated this pattern:

monoids       2d TQFTs
monoidal categories 3d TQFTs
monoidal 2-categories? 4d TQFTs?

in which increase in dimension goes along with categorification

Baez & Dolan (1995) — considered \((n+k)\)-categories with only one object, one morphism, ..., one \((k-1)\)-morphism

& arbitrary from then on. Such a thing is an \(n\)-category with extra structure. E.g. a 1-category with just
One object \((n=0, k=1)\) is a special sort of 0-category (set), namely a monoid. In general we call these "\(k\)-tuply monoidal \(n\)-categories":

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