Quantum Gravity Seminar
Homework 4

Alex Hoffnung

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Any $H : T^*M \to \mathbb{R}$ gives the Hamiltonian vector field

$$v_H \;=\; \frac{\partial H}{\partial p_i}\frac{\partial}{\partial q^i} \;-\; \frac{\partial H}{\partial q^i}\frac{\partial}{\partial p_i}$$

We have a 2-form ω on T^*M:

$$\omega \;=\; dq^i \wedge dp_i$$

Show

$$dH \;=\; \omega(v_h, -)$$

$$\begin{aligned}
\omega(v_H, -) &= dq^i(v_H)dp_i - dp_i(v_H)dq^i \\
&= dq^i\left(\frac{\partial H}{\partial p_i}\frac{\partial}{\partial q^i} - \frac{\partial H}{\partial q^i}\frac{\partial}{\partial p_i}\right)dp_i - dp_i\left(\frac{\partial H}{\partial p_i}\frac{\partial}{\partial q^i} - \frac{\partial H}{\partial q^i}\frac{\partial}{\partial p_i}\right)dq^i \\
&= \left(\frac{\partial H}{\partial p_i}dq^i\frac{\partial}{\partial q^i} - \frac{\partial H}{\partial q^i}dq^i\frac{\partial}{\partial p_i}\right)dp_i - \left(\frac{\partial H}{\partial p_i}dp_i\frac{\partial}{\partial q^i} - \frac{\partial H}{\partial q^i}dp_i\frac{\partial}{\partial p_i}\right)dq^i \\
&= \frac{\partial H}{\partial p_i}dp_i + \frac{\partial H}{\partial q^i}dq^i \\
&= dH
\end{aligned}$$