

QG Homework 1
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1

$$\begin{aligned}
& (((\lambda f.\lambda x.f(f(x))))(\lambda g.\lambda x.g(g(x))))(\lambda z.z+1))(0) \\
&= (((\lambda f.\lambda x.f(f(f(x))))(\lambda g.\lambda y.g(g(y))))(\lambda z.z+1))(0) \\
&= ((\lambda x.(\lambda g.\lambda y.g(g(y)))(\lambda g.\lambda y.g(g(y)))(\lambda g.\lambda y.g(g(y))(x))))(\lambda z.z+1))(0) \\
&= ((\lambda x.(\lambda g.\lambda y.g(g(y)))(\lambda g.\lambda y.g(g(y)))(\lambda y.x(x(y)))))(\lambda z.z+1))(0) \\
&= ((\lambda x.(\lambda g.\lambda y.g(g(y)))(\lambda g.\lambda y.g(g(y)))(\lambda u.x(x(u)))))(\lambda z.z+1))(0) \\
&= ((\lambda x.(\lambda g.\lambda y.g(g(y)))(\lambda y.(\lambda u.x(x(u)))(\lambda u.x(x(u))(y)))))(\lambda z.z+1))(0) \\
&= ((\lambda x.(\lambda g.\lambda y.g(g(y)))(\lambda y.(\lambda u.x(x(u)))(x(x(y)))))(\lambda z.z+1))(0) \\
&= ((\lambda x.(\lambda g.\lambda y.g(g(y)))(\lambda y.x(x(x(y)))))(\lambda z.z+1))(0) \\
&= ((\lambda x.(\lambda g.\lambda y.g(g(y)))(\lambda v.x(x(x(v)))))(\lambda z.z+1))(0) \\
&= ((\lambda x.(\lambda y.(\lambda v.x(x(x(v)))))(\lambda v.x(x(x(v))))(y))))(\lambda z.z+1))(0) \\
&= ((\lambda x.(\lambda y.(\lambda v.x(x(x(v)))))(x(x(x(y)))))))(\lambda z.z+1))(0) \\
&= ((\lambda x.(\lambda y.x(x(x(x(x(y)))))))))(\lambda z.z+1))(0) \\
&= (\lambda y.(\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)((\lambda z.z+1)(y))))))))(0) \\
&= (\lambda y.(\lambda z.z+8)(y))(0) \\
&= (\lambda z.z+8)(0) \\
&= 0 + 8 \\
&= 8
\end{aligned}$$

2

$$\omega = \lambda x.x(x)$$

What is $\omega(\omega)$?

$$\omega(\omega) = (\lambda x.x(x))(\lambda x.x(x)) = (\lambda x.x(x))(\lambda x.x(x)) = \omega(\omega)$$

So, $\omega(\omega)$ is an infinite recursion which doesn't have a base case.