

# Quantum Gravity Seminar Homework – Week 0

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The exercises for this homework come from Peter Selinger's *Lecture Notes on the Lambda Calculus*, available on the web at <http://www.mscs.dal.ca/~selinger/papers/lambdanotes.pdf>.

**Exercise 1.1.** Evaluate the lambda-expression

$$(((\lambda f.\lambda x.f(f(f(x))))(\lambda g.\lambda y.g(g(y))))(\lambda z.z + 1))(0).$$

The value of the lambda-expression  $\lambda f.\lambda x.f(f(f(x)))$  is a function we may call *thrice* that takes its argument  $f$  (a function) and returns the function  $f \circ f \circ f$ . Similarly,  $\lambda g.\lambda y.g(g(y))$  is a function we'll call *twice*. The expression  $\lambda z.z + 1$  is just the function, *inc* ("increment"), that returns its argument with 1 added to it. Our lambda-expression can then be written

$$((\textit{thrice}(\textit{twice}))(\textit{inc}))(0).$$

Applying *thrice* to its argument, we may write

$$((\textit{twice} \circ \textit{twice} \circ \textit{twice})(\textit{inc}))(0).$$

The function to be applied to 0 is  $\textit{twice}(\textit{twice}(\textit{twice}(\textit{inc})))$ , and so we may evaluate this expression and write

$$(\textit{inc} \circ \textit{inc} \circ \textit{inc} \circ \textit{inc} \circ \textit{inc} \circ \textit{inc} \circ \textit{inc} \circ \textit{inc})(0).$$

Thus, 0 is to be incremented eight times. The result is 8.

**Exercise 1.2.** Let  $\omega = \lambda x.x(x)$ . What is  $\omega(\omega)$ ?

This one is interesting. If we substitute the definition of  $\omega$  into  $\omega(\omega)$ , we get

$$(\lambda x.x(x))(\lambda x.x(x)).$$

Now, we can replace the dummy variable  $x$  in the first lambda-expression with the argument, which is  $\lambda x.x(x)$ . The result is

$$(\lambda x.x(x))(\lambda x.x(x))!$$

So, evidently, the expression  $\omega(\omega)$  does not evaluate to any simpler expression!