10.2.07:

- Geometric Representation Theory:
  - Representations of transformation groups,
    (rep. = group action on say vector space)
  - Typically we mean linear groups on R^n space,
  - But we are interested in other transformation groups.
  - We unit groups that preserve structure

Example:
1. Diffeomorphism group → preserve differential structure

Also: Every transformation group is the group of
"something" morphism. For some unique something.
I.e. Diffeomorphism are automorphism (or symmetry)
that preserve diffeomorphism.

How do we define the structure? Given group G
the set G is acting on, we should be able to give
the structure "preserved."

Symmetry: structure is like a dual. (Symmetry is like "neg."
  aspect of structure)

Structure

Entropy

Information

Invariance

Logic: has the job of putting structure on a set.
  → Axiomatic theory puts structure on a set:
  a) Types 1) Abstract predicates
     2) axioms about predicates

Example: Euclidean geometry: { pts, lines, 2}
A point lies on a line.
A model of the theory is a concrete realization
of abstract types, predicates, axioms.
Transformation group: axiomatic theory and the same thing.

To find structure we need to look at the action of $G$ on $S$; figure out what is invariant in the group. These invariants give the abstract (isomorphic) group.

- "Orbi-simplex" picture of a transformation group. $G \leq S$!

Fix every axiomatic theory is there a transformation group? Sort of...

Restrict to Finite transformation group. ($G, S$ are finite).

Complete axiomatic theory with an axiom stating that the "semivolume" of the model is bounded by $N$.

For true type of theory, all models are isomorphic (category). The automorphism group of a model is a transformation group. This is example of group $\rightarrow$ Axiomatic theory.

Now the orbi-simplex picture is the orbit space of the action of $G$ on the simplex whose vertices are the elements of $S$.

Simplices: 0 - 0 simplex

- 1 - simplex

\[ \begin{array}{c}
\text{2} \\
\text{3 - simplex}
\end{array} \]

As long as $S$ is 3 pts we can keep objects on the board.
So let $G$ be the 3 elt group. If $G$ is a subgroup of $S_3 \times \{1\}$

Define $G$ to be the transformations that are invertible of the set $\{A, B, C\}$ that preserve $A$.

So construct a simplex of elts of $S_3 \times \{1\}$ if $|S| = n$ then\[\text{claim the n-simplex}\]

This is a hyperpolyhedral object

The orbit space is the quotient space by identifying points that are in the same orbit.

So the orbit simplex is a hyperpolyhedral space.

Basicatric subdivision: more crosses give more elts of the group

The lattice corresponds to a Young tableau: Young diagram

Note that $m \times n = n \times m$

Rule: Split: shades

Faces: 6 sufaces correspond to
Eilenberg is an acting. For the full group: