We have been talking about the category of permutation representations of $G$ over the rig $\mathbb{N}$

$$\text{Perm } G\text{-Rep}_{\mathbb{N}}$$

II

$$G\text{-Set} \xrightarrow{\text{Hecke Operators}} G\text{-Set}$$

JB described a precursor to this, which can be decategorified to $\text{Perm } G\text{-Rep}_{\mathbb{N}}$

$$\text{G-Set} \xrightarrow{\text{Span map}} \text{Perm } G\text{-Rep}_{\mathbb{N}}$$

Last time JB gave a theorem, which we need to fix.

$$\text{G-Set iso classes of span} \xrightarrow{\text{Hecke op}} \text{G-Set} = \text{Perm } G\text{-Rep}_{\mathbb{C}} \xrightarrow{\text{G-Rep}_{\mathbb{C}}}$$
A, B  G-sets

\[ \text{Span} (A, B) \quad \text{II} \quad \text{Hom} (R(A), R(B)) \]

Free \( \mathbb{N} \)-module on something

This was wrong!

Let's look at a counterexample.

Consider the terminal G-set, \( 1 \).

We were looking at "G-invariant spans".

Our decategorification looks like

\[ \text{Span} (A, B) \quad \text{II} \quad \text{Hom} (R(A), R(B)) \]

"Take iso classes"

\[ \text{Span}_{iso} (A, B) \]

"Tensor with \( \mathbb{C} \)"

\[ \text{Hom} (R(A), R(B)) \]
Categorification & Decategorification

Let's start with decategorification, which is a destructive simplification process. Thus, it is the easier of the two. Categorification requires some creativity.

\[
\begin{align*}
\text{CAT} & \quad \longrightarrow \quad \text{SET} \\
\text{FINSET} & \quad \longrightarrow \quad \text{IN} \\
\text{FD VSP} & \quad \longrightarrow \quad \text{IN}
\end{align*}
\]

"Cardinality" \\
"Free" \\
"Dimension"

\[
\begin{align*}
\text{CAT} & \quad \longrightarrow \quad \text{SET} \\
\text{SPACE} & \quad \overset{\pi_0}{\longrightarrow} \quad \text{SET} \\
\text{SPACE} & \quad \overset{\pi_1}{\longrightarrow} \quad \text{TRUTH}
\end{align*}
\]

"Take set of components" \\
"Take set of objects" \\
"Inhabitedness" \\
"\exists" turns a logical formula into a logical statement \\
"\forall" values
Here is another example of decategorification:

\[ \text{AB CAT} \xrightarrow{\text{"Grothendieck Group"}} \text{AB GRP} \]

\[ \text{GPD} \xrightarrow{\text{SPAN}} \xrightarrow{\text{"Degroupoidification"}} \text{VSP}_C \]

little; a little; fears... they bite!