Lecture 2 Jim Dolan

Geometric Representation Theory

A group equipped with an action by transformation groups is a representation.

Representations can arise as groups of transformations preserving some structure.

Ex: Diffeomorphism group / Differentiable structure

"Every transformation group is the group of something-o-morphisms for some something"

- and this something is essentially unique.

Groups describe symmetries, which are in some sense dual to structure.

Symmetry is a negative description, whereas structure is a more positive description

symmetry	structure
entropy	information
relativity	invariance

It is the job of logic to put some kind of structure on a set.

"Axiomatic Theory"

Types

Abstract Predicates

Axioms about the predicates

Ex. Euclidean geometry

point, line

"point P lies on line L"

distinct points lie on a unique line

"Model of the theory" is "a concrete realization of abstract types, predicates, & axioms"
– the Euclidean plane, for example.

"Orbi-simplex" picture of a transformation group $G \subseteq S$!

We impose some strong conditions for now
– finitary transformation group
(group & set being acted on are both finite)

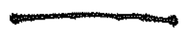
– complete axiomatic theory with an axiom stating that the "universe" of the model is bounded by N

(When we remove completeness, we switch from a group to a groupoid.)

So the orbi-simplex picture is

"the orbit space of the action of G on the simplex whose vertexes are the elements of S."

0 - simplex 

1 - simplex 

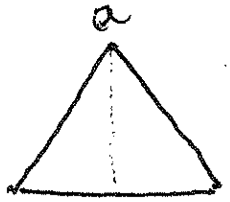
2 - simplex 

3 - simplex 

⋮

Let's look at an example with $G \subseteq 3!$

G = invertible transformations of $\{a, b, c\}$ that preserve a



We have introduced a new vertex here which we would like to label.