"Hecke Operator"

We want to use Hecke operators to find irreducible representations inside of bigger representations,

Thm:
G finite group, R doubly transitive permutation rep of G

Then R is the direct sum of 2 irreps, one of which is the 1-dim trivial rep.

$G \to S!$ induces $G \to [S^2]!$, which has only two orbits, one of which is the diagonal.

"Schur's Lemma" + "Maschke's Theorem"

"The irreps of a finite group G form an orthonormal basis for the 2-Hilbert space of fin-dim reps of G"

We need an "inner product" for this 2-Hilbert space

$$\mathrm{Hom}(R, R')$$

which gives a vector space, which is like a categorified number.

Shur's lemma then says that if you Hom an irrep into an isomorphic irrep, you get a 1-dim rep. If you Hom an irrep into a different irrep, you get 0.

$\mathrm{Hom}(R,R) = k^2$ a 2-dim vector space, since R is doubly transitive.

$1^2 + 1^2 = 2$

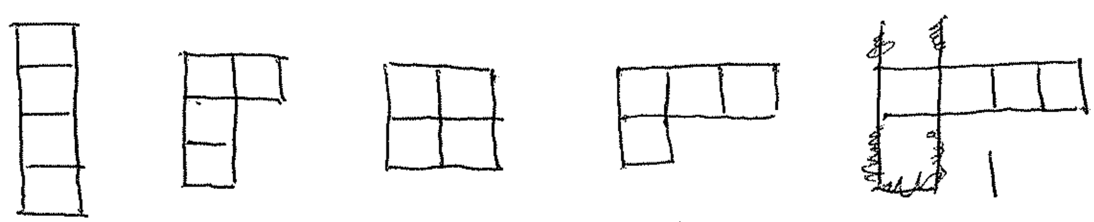
Another example:

"Find all the irreps of 4!"

We are going to use

"Gram-Schmidt Orthonormalization"

Consider all combed 4-box Young diagrams



24 12 6 4 1

Hom(V, W)

W \ V					
	24	12	6	4	1
	12	7	4	3	1
	6	4	3	2	1
	4	3	2	2	1
	1	1	1	1	1

HW: Calculate 7 relations in , .