

5/9/02

Categorified Gauge Theory

$$F = E \wedge dt + B$$

\downarrow space/time part
 \downarrow space/space part

- categorification: replace identities w/ isomorphisms
- Lie group - 1 object w/ many morphisms from it to itself that are invertible
- kers - all morph. whose source is identity in L_0 , namely 0.

Internalization

Take the defn. of group, and write it using commut. diagrams.

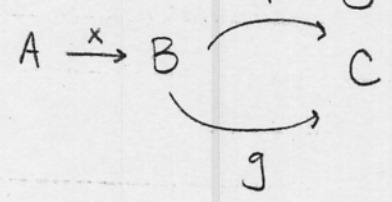
- a set G
 - a function $m: G \times G \rightarrow G$
 - an identity element $1 \in G$
- used product! so to generalize this concept to another category, we'd need it to have products
 bad!

So we think of this as $i: 1 \rightarrow G$ (morphism)

- terminal obj in Set
(1-e.l.set)
- "walking element"

In category theory: " $\forall x f(x) = g(x)$ " isn't allowed
 but " $f = g$ " is.

Also okay: $\forall x, f \circ x = g \circ x$ where



So " $f = g$ " $\Leftrightarrow \forall x, f \circ x = g \circ x$ where x is a generalized elt.
 (including $1: B \rightarrow B$ "universal elt")

(Now - $f \circ x$ is a generalized elt of C)

Note: products \dot{a}_i , terminal objects are both products! (binary \dot{a}_i , nullary)

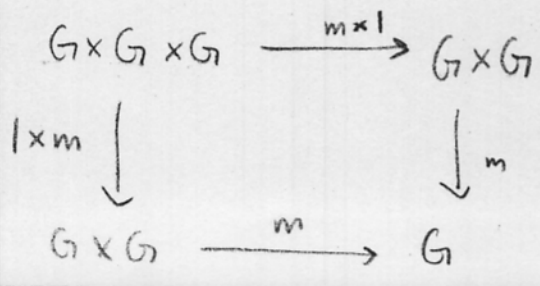
Prop: A category has all products iff it has binary products and nullary products, i.e. terminal objects.

• identity of the group = multiplying no things

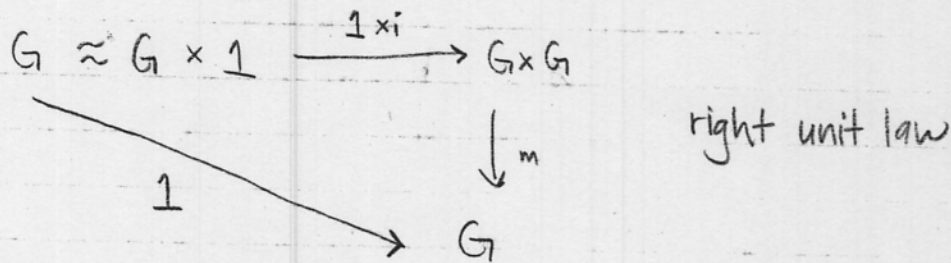
con't defn of grp

• inverses $\text{inv}: G \rightarrow G$

st. assoc law:

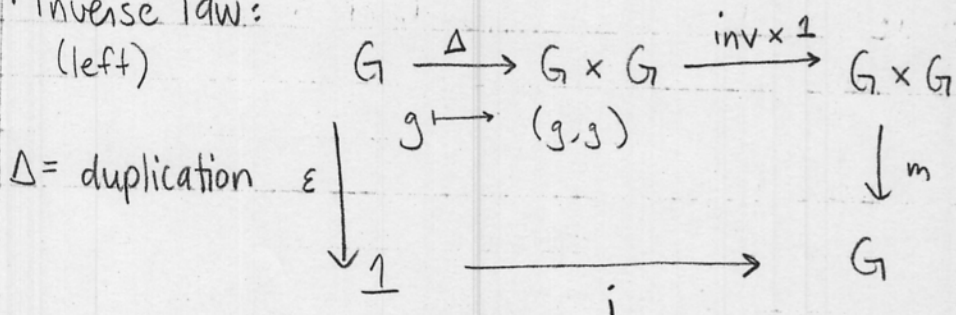


• l/r unit laws:

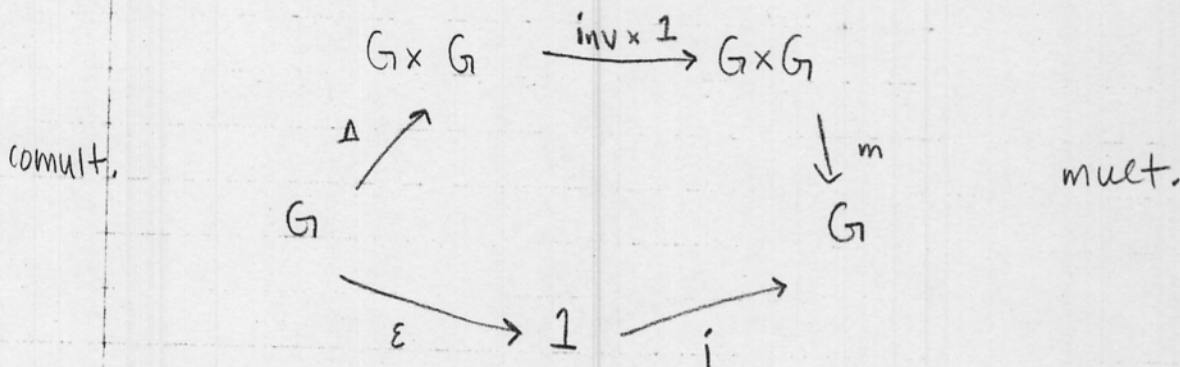


• product of object w/ terminal object is iso to object.

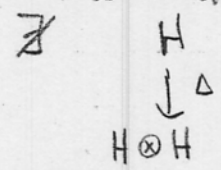
• inverse law:
(left)



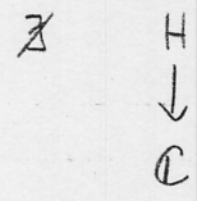
• send G into terminal object (counit)



Ex) . cat. of Hilbert spaces ^{Hilb} has a tensor product but aren't able to duplicate:



and we aren't able to delete.



So - we can define a group object in any category w/ finite products.

want to show $(g^{-1})^{-1} = g$. ie) show diagram commutes:

