5/9/02 **Categorified Gauge Theory**

$$F = \underbrace{E \wedge dt}_{\text{space/time part}} + \underbrace{B}_{\text{space/space part}}$$

- categorification: replace identities w/ isomorphisms
- Lie group — 1 object w/ many morphisms from it to itself that are invertible
- kers - all morp. whose source is identity in L_0, namely 0.

Internalization

Take the defn. of group, and write it using commut. diagrams.

- a set G
- a function $m: \underbrace{G \times G}_{\text{used product!}} \longrightarrow G$

used product! so to generalize this concept to another category, we'd need it to have products

- an identity element $\underset{\uparrow \text{bad!}}{1} \in G$

so we think of this as $i: \underline{1} \longrightarrow G$ (morphism)

- terminal obj in Set (1-elt set) "walking element"

In category theory: "$\forall x\ f(x)=g(x)$" isn't allowed
but "$f=g$" is.

Also okay: $\forall x,\ f\circ x = g\circ x$ where

$$\begin{tikzcd} A \arrow[r, "x"] & B \arrow[r, bend left, "f"] \arrow[r, bend right, "g"'] & C \end{tikzcd}$$

So "$f=g$" $\Leftrightarrow$ $\forall x,\ f\circ x = g\circ x$ where x is a generalized elt.
(including $\underline{1}: B \to B$ "universal elt")

(Now – $f\circ x$ is a generalized elt of C)

Note: products & terminal objects are both products! (binary & nullary)

Prop: A category has all products iff it has binary products and nullary products, i.e. terminal objects.

· identity of the group = multiplying no things

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con't defn of grp

· inverses $\mathrm{inv}: G \longrightarrow G$

s.t. assoc law:

$$\begin{tikzcd} G\times G\times G \arrow[r, "m\times 1"] \arrow[d, "1\times m"'] & G\times G \arrow[d, "m"] \\ G\times G \arrow[r, "m"] & G \end{tikzcd}$$

- l/r unit laws:

$$\begin{tikzcd} G \approx G \times \underline{1} \arrow[r, "1 \times i"] \arrow[dr, "\underline{1}"'] & G \times G \arrow[d, "m"] \\ & G \end{tikzcd}$$

right unit law

- product of object w/ terminal object is iso to object.

- inverse law:

(left)

$$\begin{tikzcd} G \arrow[r, "\Delta"] \arrow[d, "\varepsilon"'] & G \times G \arrow[r, "inv \times 1"] & G \times G \arrow[d, "m"] \\ \underline{1} \arrow[rr, "i"'] & & G \end{tikzcd}$$

$g \mapsto (g, g)$

Δ = duplication

- send G into terminal object ($\overset{\text{use}}{\text{counit}}$)

comult.

$$\begin{tikzcd} & G \times G \arrow[r, "inv \times 1"] & G \times G \arrow[d, "m"] \\ G \arrow[ur, "\Delta"] \arrow[dr, "\varepsilon"'] & & G \\ & \underline{1} \arrow[ur, "i"'] & \end{tikzcd}$$

mult.

Ex) · Cat. of Hilbert spaces <u>Hilb</u> has a tensor product but aren't able to duplicate:

$$\begin{tikzcd}
\nexists & H \arrow[d, "\Delta"] \\
& H \otimes H
\end{tikzcd}$$

and we aren't able to delete.

$$\begin{tikzcd}
\nexists & H \arrow[d] \\
& \mathbb{C}
\end{tikzcd}$$

So - we can define a <u>group object</u> in any category w/ finite products.

Want to show $(g^{-1})^{-1} = g$. ie) show diagram commutes:

$$\begin{tikzcd}
G \arrow[d, "inv"] \arrow[dd, bend right=60, "1"'] \\
G \arrow[d, "inv"] \\
G
\end{tikzcd}$$