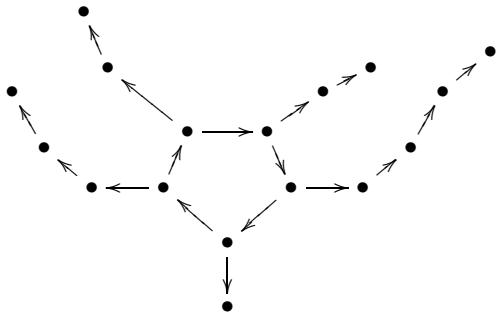
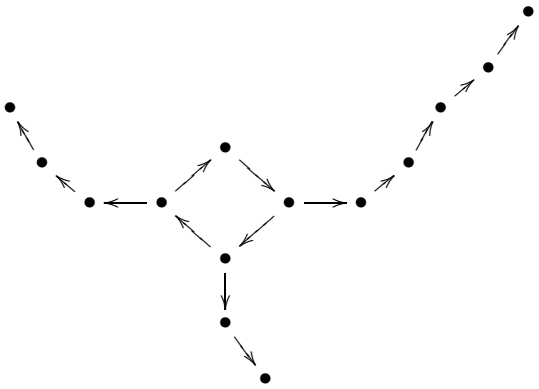
Being an Octopus[1]

John C. Baez, April 27, 2004

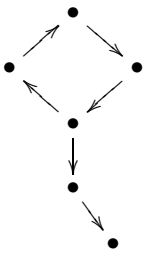
In their book **Combinatorial Species and Tree-Like Structures**, *Bergeron, Labelle and Leroux discuss a structure type* Oct, *called* **"being an octopus"**. *Instead of defining it, I'd like you to guess it from some hints, and then work out its generating function. Here is a way to put an octopus structure on a 17-element set:*



Here is a way to put an octopus structure on a 14-element set:

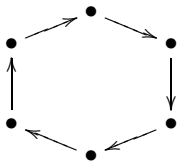


Here is one way to put an octopus structure on a 6-element set:



[1]The philosopher Heidegger wrote a book called **Being and Time**. The philosopher Sartre wrote a book called **Being and Nothingness**. Once I too wanted to be a philosopher; here is my pathetic attempt to follow in their footsteps.

and here is another:



Recall that given stuff types F and G, there is a stuff type $F \circ G$ such that to put $F \circ G$-stuff on a finite set S, we write S as a disjoint union $S_1 + \cdots + S_n$, put F-stuff on the set $\{1, \ldots, n\}$, and put G-stuff on each of the sets S_i. We have

$$|F \circ G| = |F| \circ |G|$$

whenever either side is a well-defined formal power series.

1. Find structure types F and G such that $\mathrm{Oct} \cong F \circ G$.

2. Work out $|F|$ and $|G|$ and use this to work out $|\mathrm{Oct}|$.

3. Find a simple explicit formula for the nth coefficient of the formal power series $|\mathrm{Oct}|(z)$. Use this to count the number of ways to put an octopus structure on an n-element set.