

I was inspired in part by the use of imaginary time in special relativity.  $ict$ , instead of  $ct$ , is taken to be the fourth component of spacetime. (From now on, I will leave out the constant  $c$ , taking  $c = 1$ .) This makes the time coordinate look more like the space coordinates; for example, the Minkowski arclength element  $ds$  is the square root of  $(dx)^2 + (dy)^2 + (dz)^2 - (dt)^2 = (dx)^2 + (dy)^2 + (dz)^2 + (idt)^2$ . In short, the Minkowski metric is the 4 dimensional Euclidean metric; it's all 1s down the diagonal, with nary a  $-1$  in sight. In general relativity, this is hardly useful, since the metric is going to be some wacky second rank tensor anyway, but I still always thought it would be nice to say the coordinates are orthonormal when  $g_{i,j} = \delta_{i,j}$  instead of  $g_{i,j} = \pm\delta_{i,j}$ .

My other inspiration was the antiunitary nature of the time reversal operator. In quantum mechanics, every operator is linear in the sense that  $A(|\psi\rangle + |\phi\rangle) = A|\psi\rangle + A|\phi\rangle$ . If you assume continuity, you can easily prove from this requirement that  $A(z|\psi\rangle)$  is always either  $zA|\psi\rangle$  or  $z^*A|\psi\rangle$  if  $z$  is a constant complex number and  $z^*$  is its complex conjugate. (If you can't find a proof of this in a quantum mechanics book, there's an easily adaptable theorem in most introductions to complex analysis.) If  $A(z|\psi\rangle)$  is  $zA|\psi\rangle$ ,  $A$  is linear; otherwise,  $A$  is antilinear. The vast majority of quantum mechanical operators are linear, but the time reversal operator  $T$  is antilinear. In fact,  $T$  is curiously like the complex conjugate operator  $K$ .  $K|\psi\rangle = |\psi\rangle^*$  (not to be confused with the adjoint  $|\psi\rangle^\dagger = \langle\psi|$ ), and this operator is clearly antilinear. If  $|\psi\rangle$  is a pure, spinless wavefunction,  $K$  and  $T$  do exactly the same thing, but  $T$  also reverses real numbers associated with time. In classical language,  $T$  reverses time, velocity, momentum, and so on, as well as the quantum mechanical wavefunction.

Putting these together, I realised that, if time is imaginary, then time reversal *is* complex conjugation, which is a much neater result than some nice metric for flat space.

This is how it works: all those things (quantum operators or classical variables) which reverse sign under time reversal are imaginary. Instead of working with  $t$ , work with  $it$ . Instead of  $p$  (momentum), use  $ip$ . Velocity is imaginary, the magnetic field is imaginary, and so on. It's well known that a consistent division between time reversed and nonreversed is possible; the equations of physics dictate what is reversed and what's not. For example, if charge density is unreversed but current density is reversed, Maxwell's equations tell us that the magnetic field and potential are reversed but the electric field and electrostatic potential are not. In my interpretation, all the time reversed things are considered imaginary.

$(x, y, z, it)$  are the preferred coordinates of an inertial frame. The conjugate momenta are  $(p_x, p_y, p_z, iE)$ , which is backwards (because  $p$ , being time reversed, should be imaginary, not energy  $E$ ). However, it's not as backwards as it seems. As every beginning quantum student knows,  $p_x = -i\hbar\partial/\partial x$ . (From now on, I will leave out the constant  $\hbar$ , taking  $\hbar = 1$ .) This means  $\partial/\partial x = ip_x$ , and similarly for the other spatial coordinates.  $\partial/\partial x$  seems to me a more suitable conjugate momentum than  $-i\partial/\partial x$ ; as a lame play on words, I call  $\partial/\partial x$  'complex conjugate momentum'. In general, the complex conjugate momentum is  $i$  times the conjugate momentum. Since  $E = i\partial/\partial t$ ,  $\partial/\partial(it)$  is  $E/i^2 = -E$ . Thus, the complex conjugate momenta of  $(x, y, z, it)$  are  $(ip_x, ip_y, ip_z, -E)$ , which are indeed  $i$  times the conjugate momenta.

I think complex conjugate momenta are more basic than conjugate momenta. For example, the commutator  $[x, p_x] = i$ , but  $[x, ip_x] = -1$ ; alternatively,  $[p_x, x] = -i$ , but  $[ip_x, x] = 1$ .  $1$  and  $-1$  are certainly more basic than  $i$  and  $-i$ . (My personal preference is that  $i$ , and  $\pi$  for that matter, should appear only in  $2i\pi$ , the period of the exponential function. The  $i$  in  $it$  is an artifact of incorrectly measuring time with real numbers.)

In classical mechanics, Hamilton's equations seem to favour conjugate momenta, since  $\dot{x} = \partial H/\partial p_x$  is preferable to  $\dot{x} = i\partial H/\partial(ip_x)$ . However, this is because we shouldn't be calculating  $\dot{x} = dx/dt$ ; we should be calculating  $dx/d(it) = \partial H/\partial(ip_x)$ . The Hamiltonian  $H$  is based on the conjugate momentum of  $-t$ , which is why its partial derivatives tell us derivatives with respect to  $t$ . In general, if  $a$  and  $b$  are coordinates with conjugate momenta  $p_a$  and  $p_b$ , you can calculate  $da/db$  by writing  $p_b$  as a function of all the coordinates (including time) and the other momenta (including  $H$ ).  $da/db = (da/dt)/(db/dt) = (\partial H/\partial p_a)/(\partial H/\partial p_b)$ ; by a general theorem of partial derivatives, this ratio is  $-\partial p_b/\partial p_a$ . (If  $b = t$ ,  $p_b = -H$ , so this is the usual formula for  $da/dt$ .) This is all well and good; but, since the momenta appear in a ratio, you can replace the conjugate momenta with the complex conjugate momenta. Then  $da/db = -\partial(ip_b)/\partial(ip_a)$ , which may not seem like much. However, remember that the complex conjugate momentum  $ip_a$  is  $\partial/\partial a$ , so  $da/db = -\partial(\partial/\partial b)/\partial(\partial/\partial a)$ , which I think is really cool. This is actually similar to the theorem used

above, that  $\partial a/\partial b = -(\partial c/\partial b)/(\partial c/\partial a)$ , so Hamilton's equations become (almost) obvious!  $dx/d(it) = -\partial(-H)/\partial(ip_x) = \partial H/\partial(ip_x)$  as before.

This paragraph contains further examples (to show things are never worse). The relativistic 4 momentum is the 1 form  $p_x dx + p_y dy + p_z dz - E dt = p_x dx + p_y dy + p_z dz + iE d(it)$ . Multiplied by  $i$ , this yields  $ip_x dx + ip_y dy + ip_z dz - Ed(it)$ , which is the relativistic complex conjugate 4 momentum. The stress tensor  $\rho dt \otimes dt + \dots$  is unchanged, but the timelike components must be multiplied by  $-i$  (so the twice timelike component  $\rho$ , the energy density, becomes  $-\rho$ ). The metric  $g_{t,t} dt \otimes dt + \dots$  has its components changed similarly, so the equations of general relativity retain their simple form. However, the timelike components will be imaginary numbers (so  $g_{i,j}$  is imaginary if either  $i$  or  $j$ , but not both, marks the time component). Maxwell's equations  $\nabla \cdot E = \rho$ ,  $\nabla \cdot B = 0$ , and  $\nabla \times E + \partial B/\partial t = 0$  become  $\nabla \cdot E = \rho$ ,  $\nabla \cdot (iB) = 0$ , and  $\nabla \times E + \partial(iB)/\partial(it) = 0$ .  $\nabla \times B - \partial E/\partial t = j$  becomes  $\nabla \times (iB) + \partial E/\partial(it) = ij$ , so Maxwell's equations have an unsightly minus sign changed to a plus sign. If the charge current 4 covector  $j_x dx + j_y dy + j_z dz - \rho dt$  is replaced by  $ij_x dx + ij_y dy + ij_z dz - \rho d(it)$  and the 4 potential  $A_x dx + A_y dy + A_z dz + \phi dt$  is replaced by  $iA_x dx + iA_y dy + iA_z dz + \phi d(it)$ , the manifestly covariant form of Maxwell's equations is unaltered.

(A word of caution: This isn't a new theory, just a new formulation.)

IF IT IS TIME REVERSED, IT IS IMAGINARY.

TIME IS IMAGINARY. TOBY COMPLEX CONJUGATE MOMENTUM EQUIVALENCE FORMULA:

$\partial/\partial q = ip$ . HAMILTON'S EQUATIONS ARE A TRIVIAL COROLLARY OF MY THEORY.

REORBIT VENUS AND VENUS WILL FOLLOW A DIFFERENT ORBIT.