

8.2 "Polar (Trigonometric) Form of Complex Numbers"

---

**Skills Objectives:**

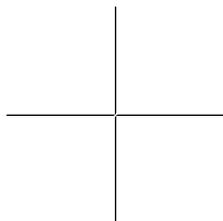
- \* Graph a point in the complex plane
- \* Convert complex numbers in rectangular form to polar form
- \* Convert complex numbers in polar form to rectangular form

**Conceptual Objectives:**

- \* Understand that a complex number can be represented in either rectangular or polar form
  - \* Relate the horizontal axis in the complex plane to the real component of a complex number
  - \* Relate the vertical axis in the complex plane to the imaginary component of a complex number
- 

## Complex Numbers in Rectangular Form

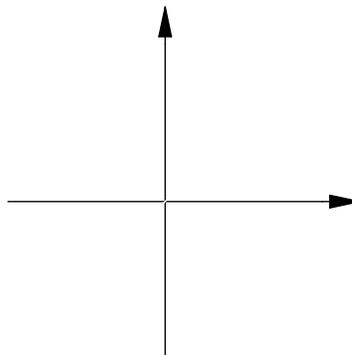
When we have complex numbers, we refer to the **standard (rectangular) form** as  $a + bi$ , where  $a$  represents the real part and  $b$  represents the imaginary part. If we let the horizontal axis be the real axis and the vertical axis be the imaginary axis, the result is the **complex plane**.



The variable  $z$  is often used to represent a complex number:  $z = x + iy$ .

Complex numbers are analogous to vectors.

For example:

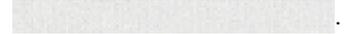


**Definition: Modulus of a Complex Number**

The modulus, or magnitude, of a complex number  $z = x + iy$  is the distance from the origin to the point  $(x, y)$  in the complex plane given by



Recall from Section 8.1 that a complex number  $z = x + iy$  has a complex conjugate



Notice that:

**Example 1:** (Finding the modulus of a complex number)

Find the modulus of:

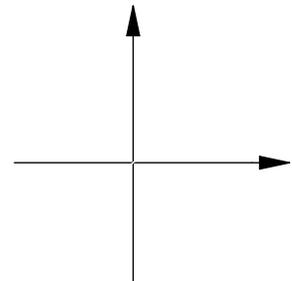
a)  $z = 3 - 2i$

b)  $z = 2 - 5i$

## Complex Numbers in Polar Form

Another convenient way of expressing complex numbers is in **polar form**. Where  $r$  represents the magnitude, or the distance from the origin to the point  $(x, y)$ , and  $\theta$  represents the direction angle.

Then we have the following relationship:



**Polar (Trigonometric) Form of Complex Numbers**

The following expression is the polar form of a complex number:

$$z = r(\cos \theta + j \sin \theta)$$

where  $r$  represents the modulus of the complex number and  $\theta$  represents the argument of  $z$ .

The following is standard notation for modulus and argument:

$$r = |z| \quad \theta = \arg z$$

**Converting Complex Numbers Between Rectangular and Polar Forms**

We can convert back and forth between rectangular and polar (trigonometric) forms of complex numbers using the modulus and trigonometric ratios.

**Converting Complex Numbers From Rectangular Form to Polar Form**

Step 1: Plot the point,  $(x, y)$ , in the complex plane (note the quadrant)

Step 2: Find  $r$  where  $r = \sqrt{x^2 + y^2}$

Step 3: Find  $\theta$  (Use  $\theta = \tan^{-1}(y/x)$  where  $x \neq 0$  and  $\theta$  is in the quadrant found in step1)

Step 4: Write the complex number in polar form:  $z = r(\cos \theta + j \sin \theta)$

**Example 2:** (Converting from rectangular to polar form)

*Express the complex number  $z = 2 + 2i$  in polar form.*

**Example 3:** (Converting from rectangular to polar form)

*Express the complex number  $z = 1 - i\sqrt{3}$  in polar form.*

**Example 4:** (Converting from rectangular to polar form)

*Express the complex number  $z = 3 - 4i$  in polar form.*

**Example 5:** (Converting from rectangular to polar form)

*Express the complex number  $z = -1 + 2i$  in polar form.*

To convert from polar to rectangular form, simply evaluate the trigonometric functions

**Example 7:** (Converting from polar to rectangular form)

*Express  $z = 2 \left( \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$  in rectangular form.*

**Example 8:** (Converting from polar to rectangular form)

*Express  $z = 4 (\cos 35^\circ + i \sin 35^\circ)$  in rectangular form.*