MATH 009B (053)
Quiz 1 Solutions

Problem 1

Evaluate the indefinite integral

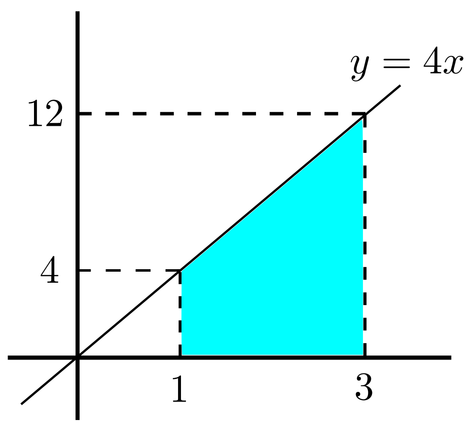
$$\int \frac{\sqrt[3]{x}}{x}\, dx$$

Solution: First simplify the integrand, and then use the power rule:

$$\begin{aligned}\int \frac{\sqrt[3]{x}}{x}\, dx = \int \frac{x^{\frac{1}{3}}}{x}\, dx = \int x^{-\frac{2}{3}}\, dx &= \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + C \\ &= 3x^{\frac{1}{3}} + C \quad \text{or} \quad 3\sqrt[3]{x} + C\end{aligned}$$

Problem 2

Evaluate the definite integral $\int_1^3 4x\, dx$ by graphing the function and using area formulas from geometry, as we did in class. (Do not use calculus.)



Solution: The graph of the function is in the figure above, with the integral being represented as the shaded region. We plug in the x-values into $y = 4x$ to get the y-values. Then we need to find the area of the region using geometry, not calculus. (Although you should know the answer via calculus since the integral of $4x$ is $2x^2$, hence the solution is $2(3^2 - 1^2) = 16$). The shaded region is a trapezoid. You can decompose it into a triangle and a rectangle, or use the area of a trapezoid directly. The values a and b are the two heights of the trapezoid, and the h is the width:

$$A_{\text{trapezoid}} = \frac{1}{2}(a+b)h = \frac{1}{2}(12+4)(3-1) = \frac{1}{2} \cdot 16 \cdot 2 = 16$$