

# MATH 009B (053)

## Quiz 3 Solutions

### Problem 1

Evaluate the following integral

$$\int \frac{x}{x^4 + 1} dx$$

Solution: This problem is a  $u$ -substitution question. Remember that if you change variables to  $u$ , you **cannot** have any  $x$  variables left in the integral. Everything must be in terms of  $u$ . For this question, we take  $u = x^2$ , which means that  $du = 2x dx$ , or alternatively  $\frac{1}{2} du = x dx$ . Now,  $x^4 = (x^2)^2 = u^2$ . Making these substitutions, we have

$$\begin{aligned} \int \frac{x}{x^4 + 1} dx &= \frac{1}{2} \int \frac{1}{u^2 + 1} du \\ &= \frac{1}{2} \arctan(u) + C \\ &= \frac{1}{2} \arctan(x^2) + C \end{aligned}$$

Note we *always* return the variable back to the one the question was asked, so the final answer is in terms of  $x$ , not  $u$ .

### Problem 2

Evaluate the following integral

$$\int x \cdot e^{x^2} dx$$

Solution: This problem is also a  $u$ -substitution question. Choose  $u = x^2$ , so then  $du = 2x dx$ , or alternatively  $\frac{1}{2} du = x dx$ . Making these substitutions, we have

$$\begin{aligned} \int x \cdot e^{x^2} dx &= \frac{1}{2} \int e^u du \\ &= \frac{1}{2} e^u + C \\ &= \frac{1}{2} e^{x^2} + C \end{aligned}$$