MATH 009B (053)
Quiz 4 Solutions

Problem 1

Evaluate the following integral

$$\int \cos^3(x)\sin^2(x)\ dx$$

Solution: This problem requires you to remember which trig function to pull out in order to make the correct substitution. In the case where one of the powers of sine or cosine is odd, we pull out one copy of the trig function with the odd power. Then we can use trig identities to convert the remaining trig functions to all cosines or all sines. For this problem, we will pull out a copy of cosine, and use the trig identity $\cos^2(x) = 1 - \sin^2(x)$. Then use the substitution $u = \sin(x)$ and $du = \cos(x)\ dx$. The computation follows

$$\begin{aligned}
\int \cos^3(x)\sin^2(x)\ dx &= \int \cos^2(x)\sin^2(x)\cos(x)\ dx \\
&= \int (1-\sin^2(x))\sin^2(x)\cos(x)\ dx \\
&= \int (1-u^2)u^2\ du \\
&= \int u^2 - u^4\ du \\
&= \frac{1}{3}u^3 - \frac{1}{5}u^5 + C \\
&= \frac{1}{3}\sin^3(x) - \frac{1}{5}\sin^5(x) + C
\end{aligned}$$

Note we *always* return the variable back to the one the question was asked, so the final answer is in terms of x, not u.

Problem 2

Evaluate the following integral

$$\int x \cdot \ln(x)\ dx$$

Solution: This is an integration by parts problem. Use the acronym LookIAteTheEgg (LIATE) to choose what the u and dv have to be. Logarithm comes before Algebraic, so we choose $u = \ln(x)$ and $dv = x\ dx$, which implies that $du = \frac{1}{x}\ dx$ and $v = \frac{1}{2}x^2$.

$$\begin{aligned}
\int u\ dv &= uv - \int v\ du \\
\int \ln(x)\cdot x\ dx &= \frac{1}{2}x^2\ln(x) - \frac{1}{2}\int x^2 \cdot \frac{1}{x}\ dx \\
&= \frac{1}{2}x^2\ln(x) - \frac{1}{2}\int x\ dx \\
&= \frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2 + C \\
&= \frac{1}{4}x^2(2\ln(x) - 1) + C
\end{aligned}$$