MATH 009B (053)
Quiz 5 Solutions

Problem 1

Determine the partial fraction decomposition of

$$\frac{x^2+1}{x(x^2+2)}$$

Solution: Recall the numerators that are required to deal with the given denominator. We have an x which has a constant numerator, say A. The second factor is an irreducible polynomial of the second order, which has numerator $Bx+C$. Therefore we have

$$\begin{aligned}\frac{x^2+1}{x(x^2+2)} &= \frac{A}{x}+\frac{Bx+C}{x^2+2}\\ &= \frac{A(x^2+2)+(Bx+C)x}{x(x^2+2)}\\ &= \frac{(A+B)x^2+Cx+2A}{x(x^2+2)}\end{aligned}$$

which gives us a way to match the numerator of the LHS and RHS. **NOTE: For the quiz, you only needed to set up the first line of the above equation.** We set up a system of equations by setting the coefficients of the x^2, x, and constant terms equal to each other:

$$\begin{aligned}A+B&=1\\ C&=0\\ 2A&=1\end{aligned}$$

By solving the system of equations, we have that $A=\frac{1}{2}, B=\frac{1}{2}$, and $C=0$. So the final decomposition is

$$\frac{x^2+1}{x(x^2+2)}=\frac{1}{2x}+\frac{x}{2(x^2+2)}$$

Problem 2

Evaluate the following integral

$$\int \frac{1}{x(x+1)^2}\,dx$$

Solution: Use the hint and the linearity of the integral to compute:

$$\begin{aligned}\int \frac{1}{x(x+1)^2}\,dx &= \int\left(\frac{1}{x}-\frac{1}{x+1}-\frac{1}{(x+1)^2}\right)\,dx\\ &= \int\frac{1}{x}\,dx-\int\frac{1}{x+1}\,dx-\int\frac{1}{(x+1)^2}\,dx\\ &= \ln(|x|)-\ln(|x+1|)+\frac{1}{x+1}+C\end{aligned}$$

The first integral is by the formula. The second and third integrals are computed using u-substitution with $u=x+1$.