

MATH 009B (053)

Quiz 6 Solutions

Problem 1

Evaluate the improper integral

$$\int_0^{\infty} \frac{e^x}{e^{2x} + 1} dx$$

Solution: The first step in these questions is to ask yourself if you can integrate the function directly. If no, you have to do some kind of comparison test. If yes, we follow the following procedure. Check both bounds of integration to see when the bounds are plugged into the integrand, if it is $\pm\infty$. Then change the appropriate bounds to some constant, say t , and rewrite as a limit:

$$\begin{aligned} \int_0^{\infty} \frac{e^x}{e^{2x} + 1} dx &= \int_1^{\infty} \frac{1}{u^2 + 1} du \quad \text{use } u = e^x \\ &= \lim_{t \rightarrow \infty} \int_1^t \frac{1}{u^2 + 1} du \\ &= \lim_{t \rightarrow \infty} \arctan(u) \Big|_1^t \\ &= \lim_{t \rightarrow \infty} \arctan(t) - \arctan(1) \\ &= \frac{\pi}{2} - \frac{\pi}{4} \\ &= \frac{\pi}{4} \end{aligned}$$