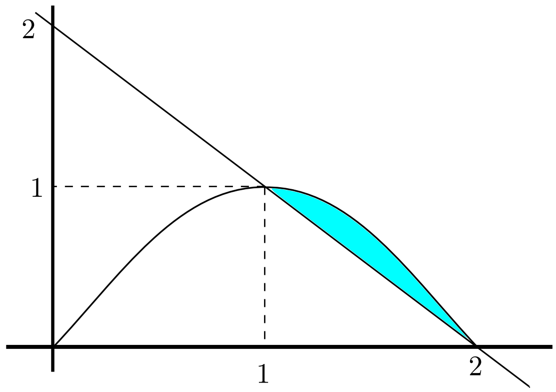
MATH 009B (053)
Quiz 7 Solutions

Problem 1

Find the area between the curves $y = 2 - x$ and $y = 2x - x^2$.

Solution: For these problems, it is helpful, but not required, to draw a picture. The graphs are given below. The area of the shaded region is what we are looking for. First, we must locate



the intersection points of the two curves. In this simple case, there is only one region, and the intersection points will be the bounds of integration. This may not always be true in the case of more complicated areas, so the picture can help. We find the bounds by setting the equations equal, and solving for x:

$$\begin{aligned}
2 - x &= 2x - x^2 \\
x^2 - 3x + 2 &= 0 \\
(x-1)(x-2) &= 0 \\
x &= 1 \text{ and } \ x = 2
\end{aligned}$$

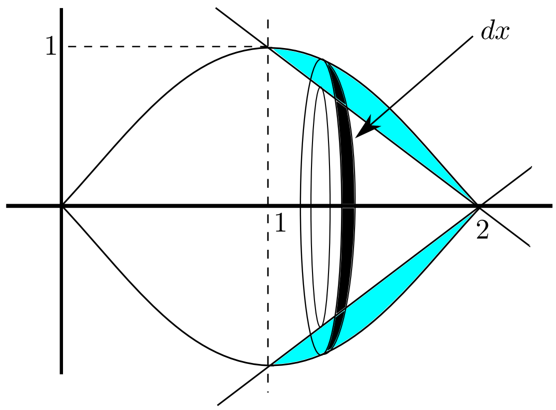
It is easy to integrate this problem with respect to the x variable, so we proceed by seeing that the top function is $f(x) = 2x - x^2$ and the bottom function is $g(x) = 2 - x$. Then just apply the area formula

$$\begin{aligned}
A &= \int_a^b (f(x) - g(x))\ dx \\
&= \int_1^2 (2x - x^2 - (2 - x))\ dx \\
&= \int_1^2 (-x^2 + 3x - 2)\ dx \\
&= \left(-\frac{1}{3}x^3 + \frac{3}{2}x^2 - 2x \right)\Bigg|_1^2 \\
&= -\frac{8}{3} + 6 - 4 + \frac{1}{3} - \frac{3}{2} + 2 \\
&= \frac{1}{6}
\end{aligned}$$

Problem 2

Find the volume of the solid obtained by revolving about the x-axis the region in the plane bounded by the curves $y = 2 - x$ and $y = 2x - x^2$.

Solution: Again, the picture can be helpful. Rotate the previous picture around the x-axis. Notice that that shaded regions do not touch each other. This is indication that washers should



be used, and the representative washer has been given. The washers would have to be moved from left to right to "shade in" The blue region, and the width of the washer is given as dx which tells us we should integrate with respect to x. The bounds of integration are the same as before, and $R(x)$ is the big radius, and $r(x)$ is the small radius:

$$\begin{aligned}
V &= \pi \int_a^b \left(R(x)^2 - r(x)^2\right)\, dx \\
&= \pi \int_1^2 \left((2x - x^2)^2 - (2 - x)^2\right)\, dx \\
&= \pi \int_1^2 x^4 - 4x^3 + 3x^2 + 4x - 4\ dx \\
&= \pi \left(\frac{1}{5}x^5 - x^4 + x^3 + 2x^2 - 4x\right)\Big|_1^2 \\
&= \frac{\pi}{5}
\end{aligned}$$