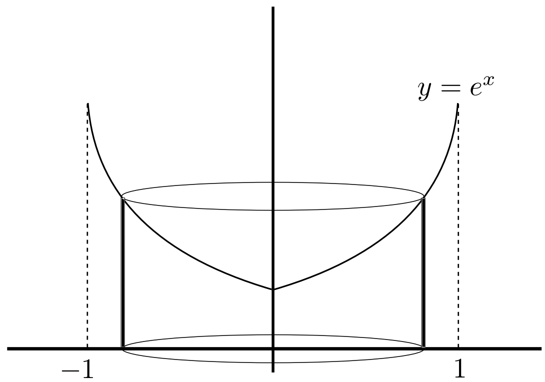
MATH 009B (053)
Quiz 8 Solutions

Problem 1

Use the shell method to find the volume of the solid obtained by revolving about the y-axis the region in the plane bounded by $y = e^x$, the x-axis, $x = 0$, and $x = 1$.

Solution: For these problems, it is helpful, but not required, to draw a picture. The graph is given below. The representative shell is drawn, and notice that the shell would have to expand to



the left and right, while moving up the function $y = e^x$ to cover the region of interest. The shell thickness is dx, which would mean we integrate with respect to x. We find the bounds by looking at the region in quadrant 1. The x values are bounded in the interval $0 \leq x \leq 1$, so these are our bounds of integration. To apply the formula, we need the radius of the shell, $r(x)$, and the height, $h(x)$. The height of the shell is $h(x) = e^x$, the function itself. Be careful with the radius of the shell, as the radius can be complicated depending on the problem. Here, we simply measure from the axis of rotation ($x = 0$) to the shell edge, which just gives us $r(x) = x$. Now compute:

$$\begin{aligned}
V &= \int_a^b 2\pi r(x)h(x)\ dx \\
&= 2\pi \int_0^1 xe^x\ dx \qquad \text{use integration by parts } u = x, dv = e^x dx \\
&= 2\pi \left(xe^x|_0^1 - \int_0^1 e^x\ dx \right) \\
&= 2\pi (xe^x|_0^1 - e^x|_0^1) \\
&= 2\pi (e - (e - 1)) \\
&= 2\pi
\end{aligned}$$