**MATH 009C** **FINAL** **August 12, 2017**

**Name:** ____________________________ **Score:** _______ / 100

**Student ID:** ____________________________

## DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| ✓ | | | | | | | | | | 200 |
| **Score** | | | | | | | | | | |
| **Pts. Possible** | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 210 |

## INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 9 question exam.
- Students have 2 hours to complete the exam.
- The test will be out of **200** points (8 questions). You may attempt a $9^{th}$ question, which will have a maximum of 10 possible points. The highest possible score is therefore **210** points.
- In the above table, the row with the ✓, is for you to keep track of the problems you are attempting/completing.
- You may complete parts of problems, as partial credit will be given based on correctness, completeness, and ideas that are leading to the correct solutions.
- **PLEASE SHOW ALL WORK. Any unjustified claims will receive no credit. This means you need to state which test you are using for series questions!** Clearly box your final answer.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- The back of the test can be used for scratch work.

GOOD LUCK!

**FORMULAS:**

| Common Taylor Series | Common Taylor Series |
|---|---|
| $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, for all $\lvert x\rvert < 1$ | $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$, for all $x \in \mathbb{R}$ |
| $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, for all $x \in \mathbb{R}$ | $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$, for all $x \in \mathbb{R}$ |
| $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$, for $x \in (-1, 1]$ | $\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$, for $\lvert x\rvert \le 1$ |
| $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$, for $\lvert x-a\rvert < R$ | $(1+x)^m = \sum_{n=0}^{\infty} \binom{m}{n} x^n$, for $\lvert x\rvert < 1$ |

1) (10 pts.) (a) Determine whether the sequence converges or diverges:

$$a_n = \frac{\cos^2(n)}{2^n}.$$

(15 pts.) (b) Determine whether the sequence converges or diverges:

$$a_n = n \sin\left(\frac{1}{n}\right).$$

2) (10 pts.) Determine whether the series is convergent or divergent:

$$\sum_{n=1}^{\infty} \arctan(n).$$

(15 pts.) (b) Determine whether the series is convergent or divergent:

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}.$$

3) (25 pts.) Determine whether the series is convergent or divergent

$$\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}.$$

4) (25 pts.) Determine whether the series is convergent or divergent

$$\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right).$$

5) (15 pts.) (a) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}.$$

(10 pts.) (b) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2}.$$

6) (25 pts.) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n^2-1}}.$$

7) (25 pts.) Find the radius of convergence and interval of convergence for the following power series (This is known as the *Bessel function of order 1*):

$$\sum_{n=0}^{\infty} \frac{(-1)^n \, x^{2n+1}}{n! \, (n+1)! \, 2^{2n+1}}$$

8) Find the sum of the following series:

( 5 pts. ) (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)!4^{2n+1}}$

( 5 pts. ) (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!6^{2n}}$

( 10 pts. ) (c) $\displaystyle\sum_{n=1}^{\infty} (-1^n) \frac{x^{4n}}{n!}$

9) ( 15 pts. ) (a) Compute the following integral using Taylor series. (*Hint: Be careful about the $n = 0$ term, you can't have 0/0.*)

$$\int \frac{e^x}{x}\, dx$$

( 10 pts. ) (b) Find the Taylor series centered at $a = 0$ for the function

$$f(x) = 2xe^{x^2}$$

**THIS PAGE IS LEFT BLANK FOR ANY SCRATCH WORK**

**END OF TEST**