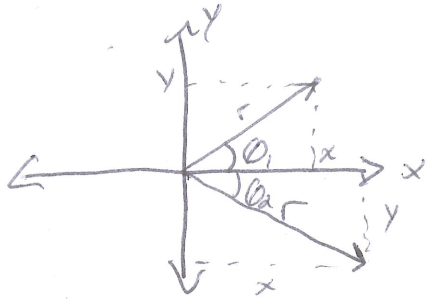
Section 9.4 - Polar Coordinates

Instead of using (x,y) coordinates in $\mathbb{R}^2$, we can use (r,θ) to describe the same point in $\mathbb{R}^2$.



Given a point (x,y) we can find the polar representation via

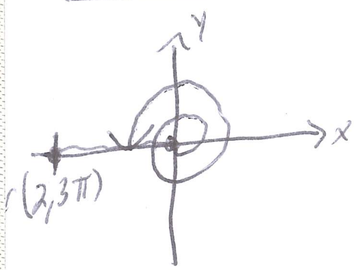
$$\boxed{r^2 = x^2 + y^2 \quad \text{and} \quad \tan\theta = y/x}$$

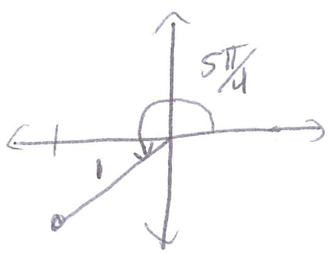
Given a point (r,θ) we can find ~~that~~ it's Cartesian representation via

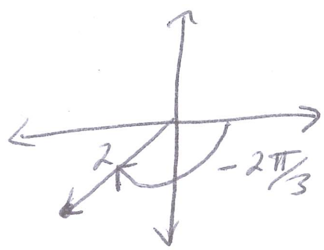
$$x = r\cos(\theta) \quad \text{and} \quad y = r\sin(\theta)$$

Note: $r \geq 0$

Plot: $(2, 3\pi)$, $(1, 5\pi/4)$, $(2, -2\pi/3)$







Convert: $(2, \pi/3)$ to Cartesian: $(1, \sqrt{3})$

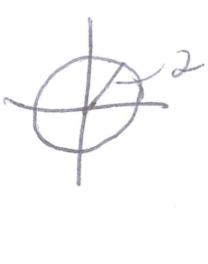
$(+1, -1)$ to polar : $(\sqrt{2}, -\pi/4)$

$(\sqrt{2}, 7\pi/4)$

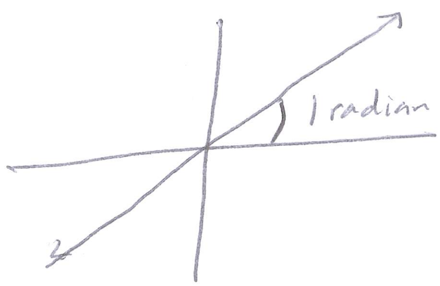
Graphs are now given as $r = f(\theta)$

Ex) $r = 2 \Rightarrow$ all points s.t. $(2,\theta)\ \forall\theta$

circle



Ex) $\theta = 1$ $(r, 1)$



Ex) $r = 2\cos(\theta)$

Plot points, or

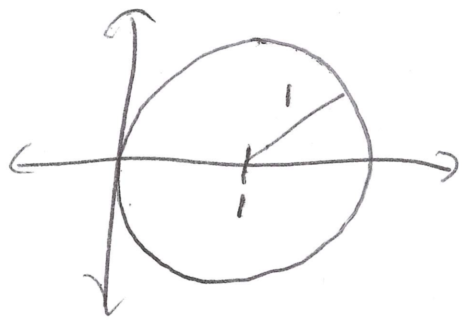
$$\cos(\theta) = \frac{r}{2}$$

$$\frac{x}{r} = \frac{r}{2}$$

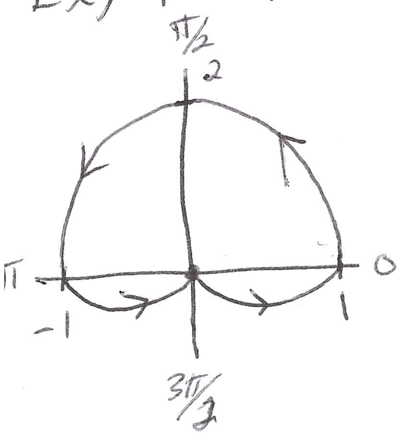
$$r^2 = 2x$$

$$x^2 + y^2 = 2x$$

$$(x-1)^2 + y^2 = 1$$



Ex) $r = 1 + \sin(\theta)$



Cardioid

Ex) $r = \cos(2\theta)$ $\quad 0 \le \theta \le 2\pi$