MATH 009C **PRACTICE FINAL** **August 12, 2017**

Name: ___________________________ **Score:** _______ / 100

Student ID: ___________________________

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

	1	2	3	4	5	6	7	8	9	Total
✓										200
Score										
Pts. Possible	25	25	25	25	25	25	25	25	25	210

INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 9 question exam.
- Students have 2 hours to complete the exam.
- The test will be out of **200** points (8 questions). You may attempt a 9^{th} question, which will have a maximum of 10 possible points. The highest possible score is therefore **210** points.
- In the above table, the row with the ✓, is for you to keep track of the problems you are attempting/completing.
- You may complete parts of problems, as partial credit will be given based on correctness, completeness, and ideas that are leading to the correct solutions.
- **PLEASE SHOW ALL WORK**. Any unjustified claims will receive no credit. This means you need to state which test you are using for series questions! Clearly box your final answer.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- The back of the test can be used for scratch work.

GOOD LUCK!

FORMULAS:

Common Taylor Series	Common Taylor Series
$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad \text{for all } \lvert x\rvert < 1$	$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, \quad \text{for all } x \in \mathbb{R}$
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad \text{for all } x \in \mathbb{R}$	$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, \quad \text{for all } x \in \mathbb{R}$
$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}, \quad \text{for } x \in (-1, 1]$	$\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}, \quad \text{for } \lvert x\rvert \leq 1$
$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n, \quad \text{for } \lvert x-a\rvert < R$	$(1+x)^m = \sum_{n=0}^{\infty} \binom{m}{n} x^n, \quad \text{for } \lvert x\rvert < 1$

1) (10 pts.) (a) Determine whether the sequence converges or diverges:

$$a_n = \frac{(2n-1)!}{(2n+1)!}.$$

(15 pts.) (b) Determine whether the sequence converges or diverges:

$$a_n = \left(1 + \frac{2}{n}\right)^n$$

2) (10 pts.) Determine whether the series is convergent or divergent:

$$\sum_{n=1}^{\infty} \sqrt[n]{2}$$

(15 pts.) (b) Determine whether the series is convergent or divergent:

$$\sum_{n=1}^{\infty} \frac{1+6^n}{7^n}.$$

3) (25 pts.) Determine whether the series is convergent or divergent

$$\sum_{n=1}^{\infty} \frac{1}{n^2+4}.$$

4) (25 pts.) Determine whether the series is convergent or divergent

$$\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$$

5) (15 pts.) (a) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!}$$

(10 pts.) (b) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$$

6) (25 pts.) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=2}^{\infty}(-1)^n\frac{n^n}{n!}$$

7) (25 pts.) Find the radius of convergence and interval of convergence for the following power series:

$$\sum_{n=0}^{\infty}(-1)^n\frac{(x-3)^n}{2n+1}$$

8) Find the sum of the following series:

(5 pts.) (a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n}$

(5 pts.) (b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n}\sqrt{3}^{2n+1}}{2n+1}$

(10 pts.) (c) $\displaystyle\sum_{n=1}^{\infty}(-1^{n})x^{2n}$

9) (20 pts.) (a) Compute the following integral using Taylor series.

$$\int \arctan(x^2)\ dx$$

(5 pts.) (b) Find the Taylor series centered at $a = \dfrac{\pi}{2}$ for

$$f(x) = \sin(x)$$

THIS PAGE IS LEFT BLANK FOR ANY SCRATCH WORK

END OF TEST