LAST NAME: FIRST NAME:

MATH 009C - Summer 2017

Quiz 1: Tuesday June 27, 2017

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1. Determine if the integral is convergent or divergent:

$$\int_2^\infty \frac{1}{x\ln(x)}\,dx$$

Solution:

Compute the integral directly using substitution $u = \ln(x)$, $du = \frac{1}{x}\,dx$

$$\begin{aligned}\int_2^\infty \frac{1}{x\ln(x)}\,dx &= \lim_{t\to\infty}\int_2^t \frac{1}{x\ln(x)}\,dx = \lim_{t\to\infty}\int_{\ln(2)}^{\ln(t)} \frac{1}{u}\,du \\ &= \lim_{t\to\infty} \ln(u)\Big|_{\ln(2)}^{\ln(t)} \\ &= \lim_{t\to\infty} \ln(\ln(t)) - \ln(\ln(2)) \\ &= \infty\end{aligned}$$

The improper integral is divergent.

2. Determine if the integral is convergent or divergent:

$$\int_1^\infty \frac{1}{\sqrt{x^6+1}}\,dx$$

Solution:

Use the (Direct or Limit) Comparison Test for Improper Integrals. Note that

$$\begin{aligned}0 < x^6 < x^6+1 \quad &\Rightarrow \quad \sqrt{x^6} < \sqrt{x^6+1} \\ &\Rightarrow \quad \frac{1}{\sqrt{x^6+1}} < \frac{1}{\sqrt{x^6}} = \frac{1}{x^3} \quad \text{for all } x > 1\end{aligned}$$

So,

$$0 < \int_1^\infty \frac{1}{\sqrt{x^6+1}}\,dx < \int_1^\infty \frac{1}{x^3}\,dx = \lim_{t\to\infty} -\frac{1}{2x^2}\Bigg|_1^t = \lim_{t\to\infty} -\frac{1}{2t^2} + \frac{1}{2} = \frac{1}{2} < \infty$$

The improper integral is convergent.

Please, show all work.

2. Eliminate the parameter for the following parameterized curve. Sketch the curve and use arrows to denote the direction.

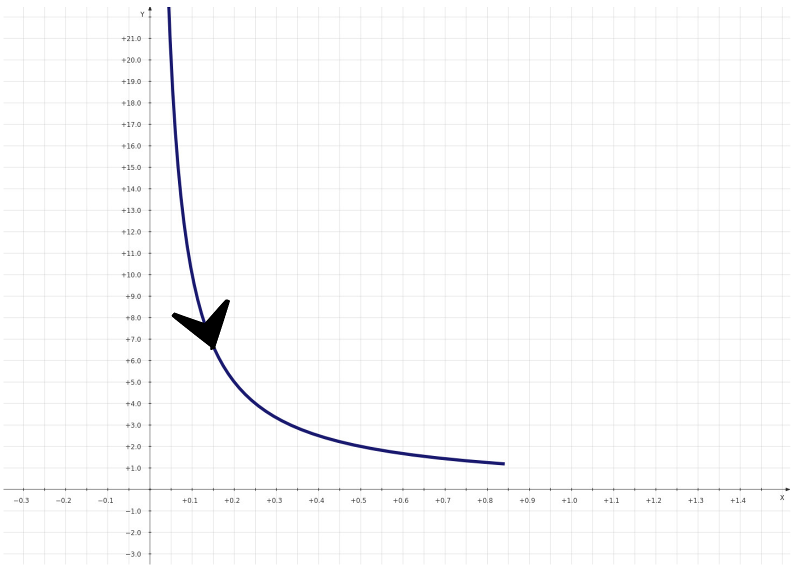
$$x = \sin(t), \quad y = \csc(t), \quad 0 < t < \frac{\pi}{2}$$

Solution:

Notice that y can be written in terms of x:

$$y = \csc(t) = \frac{1}{\sin(t)} = \frac{1}{x}$$

The restriction of $0 < t < \pi/2$ gives the restrictions: $0 < x < 1$ and $y > 1$, since $x = \sin(t)$ and $y = \csc(t)$. So then the graph is just the function $y = \frac{1}{x}$ with the above restrictions. Note that graph stops as the graph is restricted.



Please, show all work.