LAST NAME: FIRST NAME:

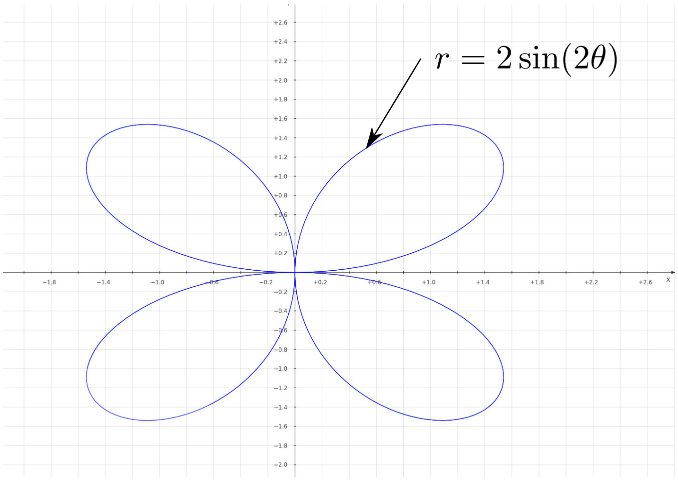
MATH 009C - Summer 2017

Quiz 3: July 11, 2017

1. Find the area of one petal of the polar rose given by:

$$r = 2\sin(2\theta)$$

Hint: The identity $\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$ *should prove useful.*



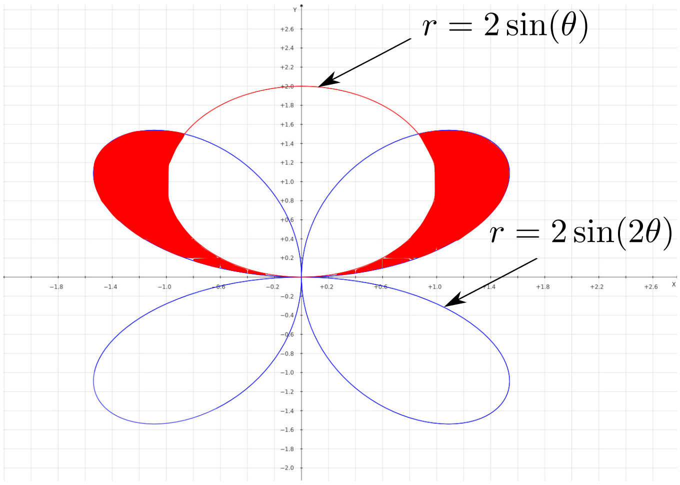
Solution: Note that the curve starts at the origin (0,0), and then at $\theta = \pi/4$, the radius is equal to 1, so the coordinate is $(1, \pi/4)$ in the first quadrant. Then, for $\theta = \pi/2$, the graph is back to the origin (0,0). So from $\theta = 0$ to $\theta = \pi/2$, the petal in the first quadrant is traced out. Now we can compute the area of the first petal:

$$\begin{aligned}
\int_a^b \frac{1}{2}(f(\theta))^2\, d\theta &= \int_0^{\frac{\pi}{2}} \frac{1}{2}(2\sin(2\theta))^2\, d\theta \\
&= \int_0^{\frac{\pi}{2}} 2\sin^2(2\theta)\, d\theta \\
&= \int_0^{\frac{\pi}{2}} (1 - \cos(2\theta))\, d\theta \\
&= \left[\theta - \frac{1}{2}\sin(2\theta)\right]_0^{\pi/2} \\
&= \frac{\pi}{2}
\end{aligned}$$

Please, show all work.

2. Find the area of the region that lies outside the circle and inside the polar rose (the region is shaded in the labeled plot below). *Hint: The identity* $\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$, $\sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$, *and* $\sin(\frac{2\pi}{3}) = -\frac{\sqrt{3}}{2}$ *should prove useful.*

$$r = 2\sin(\theta) \qquad r = 2\sin(2\theta)$$



Solution:

Find the intersection points first! We solve the following:

$$\begin{aligned}
2\sin(\theta) &= 2\sin(2\theta) \\
2\sin(\theta) &= 4\sin(\theta)\cos(\theta) \\
2\sin(\theta)\cos(\theta) - \sin(\theta) &= 0 \\
\sin(\theta)(2\cos(\theta) - 1) &= 0 \\
\sin(\theta) = 0 \quad \text{and} \quad & \cos(\theta) = \frac{1}{2} \\
\Rightarrow \quad & \theta = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}
\end{aligned}$$

Now, we can find the area of the shaded region on the left, and multiply by 2 due to symmetry. The "outside" region is the polar rose, and the "inside" region is the circle. So we integrate only to the first intersection point, ie. 0 to $\frac{\pi}{3}$. So we have

$$\begin{aligned}
A &= 2\int_a^b \frac{1}{2}\left(f(\theta)^2 - g(\theta)^2\right) d\theta = 2\int_0^{\frac{\pi}{3}} \frac{1}{2}\left((2\sin(2\theta))^2 - (2\sin(\theta))^2\right) d\theta \\
&= 4\int_0^{\frac{\pi}{3}} \left(\sin^2(2\theta) - \sin^2(\theta)\right) d\theta \\
&= 4\int_0^{\frac{\pi}{3}} \left(\left(\frac{1}{2}(1 - \cos(4\theta))\right) - \left(\frac{1}{2}(1 - \cos(2\theta))\right)\right) d\theta \\
&= 4\int_0^{\frac{\pi}{3}} (\cos(2\theta) - \cos(4\theta))\, d\theta \\
&= 4\left[\frac{1}{2}\sin(2\theta) - \frac{1}{4}\sin(4\theta)\right]_0^{\frac{\pi}{3}} = \frac{3\sqrt{3}}{2}
\end{aligned}$$