Last Name: First Name:

MATH 009C - Summer 2017

Quiz 6: August 1, 2017

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1. Determine if the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(2n)!}{2^n(n!)}$$

Solution: Note the factorials. Use the Ratio Test.

$$\begin{aligned}
\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| &= \lim_{n\to\infty} \frac{(2(n+1))!}{2^{n+1}(n+1)!} \cdot \frac{2^n(n!)}{(2n)!} \\
&= \lim_{n\to\infty} \frac{(2n+2)(2n+1)((2n)!)}{(2n)!} \cdot \frac{2^n}{2^{n+1}} \cdot \frac{n!}{(n+1)(n!)} \\
&= \lim_{n\to\infty} \frac{(2n+1)(2n+1)}{(2n+2)} \\
&= \lim_{n\to\infty} 2n+1 \\
&= \infty
\end{aligned}$$

Therefore, by the Ratio Test, the series is divergent.

Please, show all work.

2. Determine if the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\ln(n)}$$

Solution: For series that are Alternating Series, check absolute convergence first. Using the fact that $\ln(n) < n$ for all $n \geq 1$, we have

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{\ln(n)} \right| = \sum_{n=1}^{\infty} \frac{1}{\ln(n)} > \sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

where we used the Comparison Test in the last inequality. So the series is *not* absolutely convergent. Then we apply the Alternating Series Test. We will show that the limit of the b_n is zero, and the sequence is decreasing. Limit:

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{\ln(n)} = 0$$

Show decreasing:

$$f(x) = \frac{1}{\ln(x)} \quad \Rightarrow f'(x) = -\frac{1}{x(\ln(x))^2}$$

which is negative for $x > 0$, hence for $x > 1$. So by Alternating Series Test is convergent. But recall, that the series was not absolutely convergent, therefore the series is **conditionally convergent**.