LAST NAME: FIRST NAME:

MATH 009C - Summer 2017

Quiz 7: August 8, 2017

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1. Find the radius of convergence and interval of convergence for the power series.

$$\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2}$$

Solution:

Use the Ratio Test.

$$\begin{aligned}
\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| &= \lim_{n\to\infty} \left|\frac{(4x+1)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(4x+1)}\right| \\
&= |4x+1| \lim_{n\to\infty} \frac{n^2}{(n+1)^2} \\
&= |4x+1| < 1 \\
&\Rightarrow \quad -\frac{1}{2} < x < 0
\end{aligned}$$

Which means the radius of convergence is $R = \frac{1}{4}$. We must also check the endpoints. Both endpoints converge, as both series are convergent p-series with $p = 2$

$$\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2} = \sum_{n=1}^{\infty} \frac{(4(0)+1)^n}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2} = \sum_{n=1}^{\infty} \frac{\left(4\left(-\frac{1}{2}\right)+1\right)^n}{n^2} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

where the second series is an alternating absolutely convergent p-series. So the interval of convergence is $[-\frac{1}{2}, 0]$.

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2. Find the Taylor series centered at zero for the following function. State the radius of convergence.

$$f(x) = x\cos\left(\frac{1}{2}x^2\right)$$

Solution: The easiest way to approach these types of problems is to attempt to use substitution via equations on the table than to use the definition and take derivatives. So, we have by substitution by starting with cosine

$$\begin{aligned}
\cos(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \\
\cos\left(\frac{1}{2}x^2\right) &= \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{1}{2}x^2\right)^{2n}}{(2n)!} \\
&= \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{4^n (2n)!} \\
x\cos\left(\frac{1}{2}x^2\right) &= \sum_{n=0}^{\infty} \frac{x^{4n+1}}{4^n (2n)!}
\end{aligned}$$

So by Theorem, since the Taylor Series for $\cos(x)$ has $R = \infty$, then this Taylor Series also has $R = \infty$.

Please, show all work.