MATH 65B **FINAL** **May 15, 2018**

Name: ______________________________ **Score:** _______ / 100

Student ID: ______________________________

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

	1	2	3	4	5	6	7	8	9	10	11	Total
✓												100
Score												
Pts. Possible	10	10	10	10	10	10	10	10	10	10	10	110

INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 11 question exam.
- Students have 2 hours and 30 minutes to complete the exam.
- The test will be out of **100** points. You may attempt as many problems or parts of problems as you would like. The highest possible score is therefore **110** points.
- In the above table, the row with the ✓, is for you to keep track of the problems you are attempting/completing.
- You may complete parts of problems, as partial credit will be given based on correctness, completeness, and ideas that are leading to the correct solutions.
- **PLEASE SHOW ALL WORK**. **Any unjustified claims will receive no credit. This means you need to state which test you are using for series questions!** Clearly box your final answer.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- The last page of the test can be used for scratch work.

GOOD LUCK!

FORMULAS:

$\dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$	$\dfrac{dy}{dx} = \dfrac{\frac{dr}{d\theta}\sin(\theta) + r\cos(\theta)}{\frac{dr}{d\theta}\cos(\theta) - r\sin(\theta)}$
$\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad \text{for all } \lvert x\rvert < 1$	$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{(2n+1)!}, \quad \text{for all } x \in \mathbb{R}$
$e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!}, \quad \text{for all } x \in \mathbb{R}$	$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{(2n)!}, \quad \text{for all } x \in \mathbb{R}$
$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \dfrac{x^n}{n}, \quad \text{for } x \in (-1, 1]$	$\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{2n+1}, \quad \text{for } \lvert x\rvert \le 1$
$f(x) = \sum_{n=0}^{\infty} \dfrac{f^{(n)}(a)}{n!}(x-a)^n, \quad \text{for } \lvert x-a\rvert < R$	$(1+x)^m = \sum_{n=0}^{\infty} \binom{m}{n} x^n, \quad \text{for } \lvert x\rvert < 1$

1) (10 pts.) Consider the parametric curve for $0 \leq t \leq 2\pi$, given by

$$x = -2 + 2\cos(t)$$
$$y = 1 - 2\sin(t)$$

(a) (3 pts.) Sketch the graph of the parametric curve and identify the direction for increasing values of t with an arrow.
(b) (4 pts.) Find the values of t where the the tangent line has a slope of 1.
(c) (3 pts.) Determine for which values of t is the curve concave up.

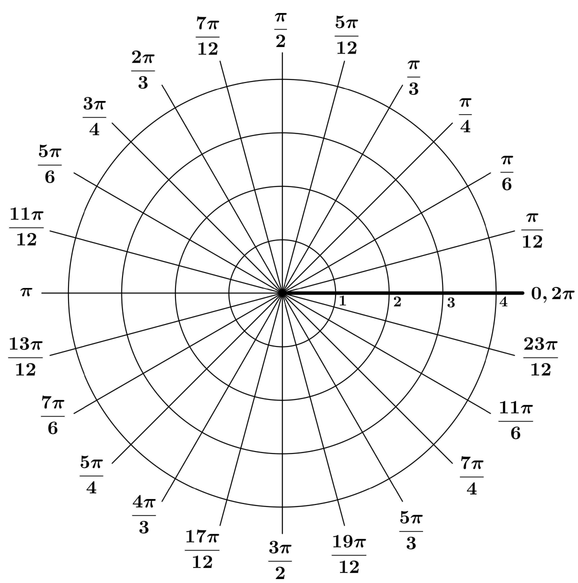
2) (10 pts.) Consider the polar curve for $0 \leq t \leq \pi$, given by

$$r = 3\cos(3\theta)$$

(a) (4 pts.) Plot the curve on the given polar grid below.

(b) (6 pts.) Find the equation of the tangent line at $\theta = \dfrac{\pi}{6}$.

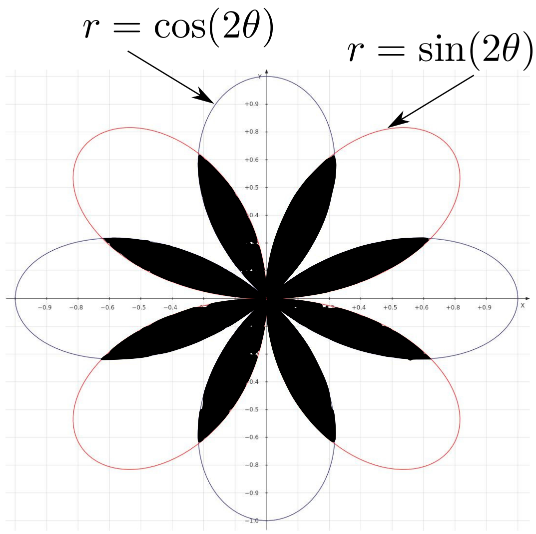
Solution:



3) (10 pts.) Find the area of the region that lies inside the polar roses (the region is shaded in the labeled plot below), for $0 \leq \theta \leq 2\pi$.

$$r = \cos(2\theta)$$
$$r = \sin(2\theta)$$

Hint: Identities that may be helpful: 1) $\sin^2(2\theta) = \frac{1}{2} - \frac{1}{2}\cos(4\theta)$*, 2)* $\cos^2(2\theta) = \frac{1}{2} + \frac{1}{2}\cos(4\theta)$.



4) (a) (5 pts.) Determine whether the sequence converges or diverges:

$$a_n = \left(\frac{1}{n}\right)^{\dfrac{1}{\ln(n)}}.$$

(b) (5 pts.) Determine whether the sequence converges or diverges:

$$a_n = \sin\left(\frac{\pi}{2} + \frac{1}{n}\right).$$

5) (a) (5 pts.) Determine whether the series is convergent or divergent:

$$\sum_{n=1}^{\infty} \arctan(n).$$

(b) (5 pts.) Determine the rational number $\frac{p}{q}$, for $p, q \in \mathbb{Z}$ which represents the decimal expansion for $0.\overline{2} = 0.22222\ldots$, using infinite series.

6) (a) (5 pts.) Determine whether the series is convergent or divergent

$$\sum_{n=2}^{\infty} \frac{1}{\ln(n)}.$$

(b) (5 pts.) Determine whether the series is convergent or divergent

$$\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right).$$

7) (a) (5 pts.) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=1}^{\infty} \frac{(\ln(n))^n}{n^n}.$$

(b) (5 pts.) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}.$$

8) (10 pts.) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt[4]{n^4 - 1}}.$$

9) (10 pts.) Find the radius of convergence and interval of convergence for the following power series. (*Note*: This is known as the *Bessel function of order 2* and is the solution to the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 4)y = 0$).

$$\sum_{n=0}^{\infty} \frac{(-1)^n \, x^{2(n+1)}}{n! \, (n+2)! \, 2^{2(n+1)}}$$

10) (10 pts.) Compute the first 4 non-zero terms of the Taylor series centered at $a = \frac{\pi}{2}$ for the following function using the definition of a Taylor series. **NOTE: You do not need to find a general formula. Write your solution as**

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3,$$

where you have to find $c_0, c_1, c_2,$ **and** c_3**.**

$$f(x) = x\sin(x)$$

11) (a) (5 pts.) Compute the following integral using Taylor series, and leave the answer as a Taylor series.

$$\int e^{-x^2}\,dx$$

(b) (5 pts.) Find the Taylor series centered at $a = 0$ for the function

$$f(x) = \frac{x - \arctan(x)}{x^2}$$

THIS PAGE IS LEFT BLANK FOR ANY SCRATCH WORK

END OF TEST