LAST NAME: KEY FIRST NAME:

MATH 65B - Spring 2018

Groupwork 1: January 16, 2018

1. Use the Fundamental Theorem of Calculus to compute the following derivative.

$$\frac{d}{dx}\int_1^{\sin(x)} \cos(t)e^{-t^3}\,dt$$

Let $F(x) = \int_1^{\sin(x)} \cos(t)e^{-t^3}\,dt$, $G(x) = \int_1^{x} \cos(t)\,e^{-t^3}\,dt$

and $g(x) = \sin(x)$. Then $g'(x) = \cos(x)$, and by FTC

$G'(x) = \cos(x)e^{-x^3}$. Note $F(x) = G(g(x))$, then

$F'(x) = G'(g(x))\cdot g'(x)$ by Chain Rule.

$\Rightarrow \boxed{F'(x) = \cos(\sin(x))\, e^{-\sin^3(x)} \cdot \cos(x)}$

2. Compute the following indefinite integral.

$$\int \sin(x)\sec^2(\cos(x))\,dx$$

Let $u = \cos(x)$, then $du = -\sin(x)dx$ (or $-du = \sin(x)dx$)

$\Rightarrow \int \sin(x)\sec^2(\cos(x))\,dx = -\int \sec^2(u)\,du$

$= -\tan(u) + C$

$= \boxed{-\tan(\cos(x)) + C}$

Please, show all work.

3. Compute the following definite integral.

$$\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx$$

Let $u = 1+2x \iff \frac{1}{2}(u-1) = x$

$du = 2dx$ or $\frac{1}{2}du = dx$

$$\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx = \int_1^9 \frac{\frac{1}{2}(u-1)}{\sqrt{u}} \cdot \frac{1}{2}du$$

Note: use $x=0$, $x=4$ with $u = 1+2x \Rightarrow u=1$, $u=9$

$$= \frac{1}{4}\int_1^9 \frac{u-1}{\sqrt{u}}\,du = \frac{1}{4}\left[\int_1^9 u^{1/2}du - \int_1^9 u^{-1/2}du\right]$$

$$= \frac{1}{4}\left[\frac{2}{3}u^{3/2}\Big|_1^9 - 2u^{1/2}\Big|_1^9\right]$$

$$= \frac{1}{4}\left[\frac{2}{3}\cdot 27 - \frac{2}{3} - 2\cdot 3 + 2\right] = \frac{1}{4}\cdot\frac{40}{3} = \boxed{\frac{10}{3}}$$

4. Compute the following limits.

(a) $\lim_{x\to 0} \frac{x}{\cos(x)}$

(b) $\lim_{x\to 1} \frac{\ln(x)}{\sin(\pi x)}$

(c) $\lim_{x\to 0^+} x^{x^2}$

(d) $\lim_{x\to\infty} x\tan\left(\frac{1}{x}\right)$

a) $\lim_{x\to 0} \frac{x}{\cos(x)} = \frac{0}{\cos(0)} = \frac{0}{1} = \boxed{0}$

b) $\lim_{x\to 1} \frac{\ln(x)}{\sin(\pi x)} = \frac{\ln(1)}{\sin(\pi)} = \frac{0}{0}$ indeterminate $\Rightarrow$ more work

Use L'Hôpital's Rule

$\Rightarrow \lim_{x\to 1} \frac{\ln(x)}{\sin(\pi x)} = \lim_{x\to 1} \frac{1/x}{\pi\cos(\pi x)} = \frac{1/1}{\pi(-1)} = \boxed{-\frac{1}{\pi}}$

c) Plugging in 0^+, we get $0^0 \Rightarrow$ indeterminate.

$$\lim_{x\to 0^+} x^{x^2} = \lim_{x\to 0^+} \exp(\ln(x^{x^2})) = \lim_{x\to 0^+} \exp(x^2\ln(x))$$

$$= \lim_{x\to 0^+} \exp\left(\frac{\ln(x)}{1/x^2}\right) = \exp\left(\lim_{x\to 0^+} \frac{\ln(x)}{1/x^2}\right) \overset{L'H}{=} \exp\left(\lim_{x\to 0^+} \frac{1/x}{-2/x^3}\right)$$

$$= \exp\left(\lim_{x\to 0^+} -\frac{1}{2}x^2\right) = \exp(0) = \boxed{1}$$

d) "Plugging in" $\infty \Rightarrow \infty \cdot \tan\left(\frac{1}{\infty}\right) = \infty \cdot 0$ indeterminate

Write $\lim_{x\to\infty} x\tan\left(\frac{1}{x}\right) = \lim_{x\to\infty} \frac{\tan(1/x)}{1/x}$ cont.

Please, show all work.

$$= \lim_{t\to 0^+} \frac{\tan(t)}{t} = \lim_{t\to 0^+} \left(\frac{1}{t}\right)\cdot\frac{\sin(t)}{\cos(t)} = \left(\lim_{t\to 0^+} \frac{\sin(t)}{t}\right)\cdot\left(\lim_{t\to 0^+} \frac{1}{\cos(t)}\right)$$

$$= 1\cdot 1 = \boxed{1}$$

5. Compute the following indefinite integral.

$$\int \frac{e^x}{1+e^{2x}}\, dx$$

This is u-substitution in disguise.

$$\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{e^x}{1+(e^x)^2}\,dx$$

Let $u = e^x$

$du = e^x\,dx$

$$= \int \frac{1}{1+u^2}\,du$$

$$= \arctan(u) + C$$

$$= \boxed{\arctan(e^x) + C}$$

6. Solve the following initial value problem.

$$\frac{dy}{dx} = 1 + y^2, \quad y(0) = 0$$

Separate the y's and x's

$$\Rightarrow \frac{dy}{1+y^2} = dx \Rightarrow \int \frac{1}{1+y^2}\,dy = \int dx$$

$$\Rightarrow \arctan(y) = x + C$$

$$\Rightarrow y(x) = \tan(x+c)$$

Use $y(0)=0$ $\Rightarrow$ $y(0) = 0 = \tan(0+c)$

$$0 = \tan(c)$$

$$\Rightarrow c = 0$$

$$\Rightarrow \boxed{y(x) = \tan(x)}$$

Check: $\frac{dy}{dx} = 1+y^2$ $\quad$ $\frac{dy}{dx} = \frac{d}{dx}\tan(x)$

$= \sec^2(x)$

$1+y^2 = 1+\tan^2(x)$

$\Rightarrow \sec^2(x) = 1+\tan^2(x)$

$\Downarrow$

identity from trig

Please, show all work.