LAST NAME: KEY FIRST NAME:

MATH 65B - Spring 2018

Groupwork 2: January 23, 2018

$\int u\,dv = uv - \int v\,du$

Choosing u:

Log

Inv Trig

Algebraic

Trig

Exp.

1. Compute the following indefinite integral.

$$\int x^6 \ln(x)\, dx$$

$u = \ln(x) \quad dv = x^6\,dx$

$du = \frac{1}{x}dx \quad v = \frac{x^7}{7}$

$$\Rightarrow \int x^6 \ln(x)\,dx = \frac{x^7}{7}\ln(x) - \int \frac{x^7}{7}\cdot\frac{1}{x}\,dx$$

$$= \frac{x^7}{7}\ln(x) - \frac{1}{7}\int x^6\,dx$$

$$= \boxed{\frac{x^7}{7}\ln(x) - \frac{1}{49}x^7}$$

$$= \boxed{\frac{1}{49}x^7(7\ln(x) - 1)}$$

2. Compute the following definite integral.

$$\int_0^{\pi} e^{\sin(x)} \cos(x)\, dx$$

Don't be fooled, this is u-substitution $\rightarrow$ easy

$u = \sin(x)$

$du = \cos(x)\,dx$

$$\int_0^{\pi} e^{\sin(x)}\cos(x)\,dx = \int_{x=0}^{x=\pi} e^u\,du$$

$$= e^u\Big|_{x=0}^{x=\pi}$$

$$= e^{\sin(x)}\Big|_0^{\pi} = e^{\sin(\pi)} - e^{\sin(0)}$$

$$= e^0 - e^0$$

$$= \boxed{0}$$

Please, show all work.

Let $u = \sin(x)$ $\quad dv = e^{2x}dx$ $\quad \int u\,dv = uv - \int v\,du$

$du = \cos(x)\,dx$ $\quad v = \frac{1}{2}e^{2x}$

3. Compute the following indefinite integral.

$$\int e^{2x}\sin(x)\,dx$$

$u = \cos(x)$ $\quad dv = e^{2x}dx$

$du = -\sin(x)dx$ $\quad v = \frac{1}{2}e^{2x}$

$$\Rightarrow \int e^{2x}\sin(x)\,dx = \frac{1}{2}e^{2x}\sin(x) - \int \frac{1}{2}e^{2x}\cos(x)\,dx$$

$$= \frac{1}{2}e^{2x}\sin(x) - \frac{1}{2}\left[\frac{1}{2}e^{2x}\cos(x) - \int \frac{1}{2}e^{2x}(-\sin(x))\,dx\right]$$

$$\int e^{2x}\sin(x)\,dx = \frac{1}{2}e^{2x}\sin(x) - \frac{1}{4}e^{2x}\cos(x) - \frac{1}{4}\int e^{2x}\sin(x)\,dx$$

$$\Rightarrow \frac{5}{4}\int e^{2x}\sin(x)\,dx = \frac{1}{2}e^{2x}\sin(x) - \frac{1}{4}e^{2x}\cos(x)$$

$$\Rightarrow \int e^{2x}\sin(x)\,dx = \boxed{\frac{2}{5}e^{2x}\sin(x) - \frac{1}{5}e^{2x}\cos(x) + C = \frac{1}{5}e^{2x}\left(2\sin(x) - \cos(x)\right) + C}$$

4. Compute the following indefinite integral.

$$\int \sin^2(x)\cos^5(x)\,dx$$

Factor out odd copy

$$\Rightarrow \int \sin^2(x)\cos^5(x)\,dx = \int \sin^2(x)\cos^4(x)\cos(x)\,dx$$

$$= \int \sin^2(x)\left(\cos^2(x)\right)^2\cos(x)\,dx$$

Let $u = \sin(x)$

$du = \cos(x)dx$

$$= \int \sin^2(x)\left(1 - \sin^2(x)\right)^2\cos(x)\,dx$$

$$= \int u^2(1-u^2)^2\,du = \int (u^6 - 2u^4 + u^2)\,du$$

$$= \frac{u^7}{7} - \frac{2}{5}u^5 + \frac{u^3}{3} + C$$

$$= \boxed{\frac{\sin^7(x)}{7} - \frac{2}{5}\sin^5(x) + \frac{\sin^3(x)}{3} + C}$$

Please, show all work.

5. Compute the following indefinite integral.

$$\int \tan^4(x)\, dx$$

$$\int \tan^4(x)\,dx = \int \tan^2(x)\tan^2(x)\,dx = \int \tan^2(x)\left(\sec^2(x)-1\right)dx$$

$$= \int \tan^2(x)\sec^2(x)\,dx - \int \tan^2(x)\,dx$$ ← use $\tan^2(x) = \sec^2(x)-1$ again

$$= \int \tan^2(x)\sec^2(x)\,dx - \int \sec^2(x)\,dx + \int dx$$

Let $u = \tan(x)$
$du = \sec^2(x)\,dx$

$$= \int u^2\,du - \int \sec^2(x)\,dx + \int dx$$

$$= \frac{u^3}{3} - \tan(x) + x + C = \boxed{\frac{\tan^3(x)}{3} - \tan(x) + x + C}$$

6. Compute the following indefinite integral.

$$\int \sin^4(x)\, dx$$

Identity

$\sin^2(x) = \frac{1}{2}(1-\cos(2x))$

$\cos^2(x) = \frac{1}{2}(1+\cos(2x))$

$$\int \sin^4(x)\,dx = \int (\sin^2(x))(\sin^2(x))\,dx$$

$$= \int \left(\tfrac{1}{2}(1-\cos(2x))\right)\left(\tfrac{1}{2}(1-\cos(2x))\right)dx$$

$$= \frac{1}{4}\int (1-\cos(2x))(1-\cos(2x))\,dx$$

$$= \frac{1}{4}\int 1 - 2\cos(2x) + \cos^2(2x)\,dx$$

$$= \frac{1}{4}\int 1 - 2\cos(2x) + \tfrac{1}{2}(1+\cos(4x))\,dx$$

$$= \frac{1}{4}\int dx - \frac{1}{2}\int \cos(2x)\,dx + \frac{1}{8}\int dx + \frac{1}{8}\int \cos(4x)\,dx$$

$$= \boxed{\frac{3}{8}x - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C}$$

Please, show all work.