Last Name: KEY First Name:

MATH 65B - Spring 2018

Groupwork 3: February 1, 2018

1. State which trigonometric substitution would be used to solve the following integrals. (*Note:* You do not need to compute the integrals for this problem.)

(a) $\int \frac{1}{x^2\sqrt{x^2+4}}\,dx \Rightarrow x = 2\tan\theta$

(b) $\int \frac{1}{\sqrt{16-x^2}}\,dx \Rightarrow x = 4\sin\theta$

(c) $\int \frac{x^3}{\sqrt{x^2-16}} \Rightarrow x = 4\sec\theta$

2. Compute the following indefinite integral.

$$\int \frac{x^3}{\sqrt{x^2+9}}\,dx$$

Use $x = 3\tan\theta$, $dx = 3\sec^2\theta\,d\theta$

$$\int \frac{x^3}{\sqrt{x^2+9}}\,dx = \int \frac{3^3\tan^3\theta}{\sqrt{9\tan^2\theta+9}}\cdot 3\sec^2\theta\,d\theta$$

$$= \int \frac{3^3\tan^3\theta}{3\sec\theta}\,3\sec^2\theta\,d\theta = 3^3\int \tan^3\theta\sec\theta\,d\theta$$

$u = \sec\theta$

$du = \sec\theta\tan\theta\,d\theta$

$$= 3^3\int \tan^2\theta\tan\theta\sec\theta\,d\theta$$

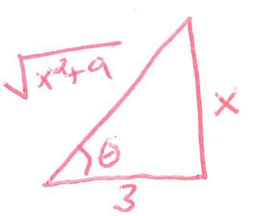
$$= 3^3\int(\sec^2\theta - 1)\tan\theta\sec\theta\,d\theta$$

$$= 3^3\int u^2-1\,du = 3^3\left(\frac{1}{3}u^3 - u\right) + C$$

$$= 3^3\left(\frac{1}{3}\sec^3\theta - \sec\theta\right) + C$$

$$= 3^3\left(\frac{1}{3}\frac{(x^2+9)^{3/2}}{3^3} - \frac{\sqrt{x^2+9}}{3}\right) + C$$

$$= \boxed{\frac{1}{3}(x^2+9)^{3/2} - 9\sqrt{x^2+9} + C}$$



$x = 3\tan\theta$

$\frac{x}{3} = \tan\theta$

$\Downarrow$

$\sec\theta = \frac{hyp}{adj}$

$\sec\theta = \frac{\sqrt{x^2+9}}{3}$

Please, show all work.

3. Compute the following indefinite integral.

$$\int \frac{\sqrt{x^2-9}}{x^3}\,dx$$

Let $x = 3\sec\theta$

$dx = 3\sec\theta\tan\theta\,d\theta$

$$\int \frac{\sqrt{x^2-9}}{x^3}\,dx = \int \frac{\sqrt{9\sec^2-9}}{3^3\sec^3\theta}\,3\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{3\tan\theta}{3^3\sec^3\theta}\,3\sec\theta\tan\theta\,d\theta$$

$$= \frac{1}{3}\int \frac{\tan^2\theta}{\sec^2\theta}\,d\theta = \frac{1}{3}\int \frac{\sin^2\theta}{\cos^2\theta}\cdot\frac{\cos^2\theta}{1}\,d\theta$$

$$= \frac{1}{3}\int \sin^2\theta\,d\theta = \frac{1}{3}\int \frac{1}{2}(1-\cos(2\theta))\,d\theta$$

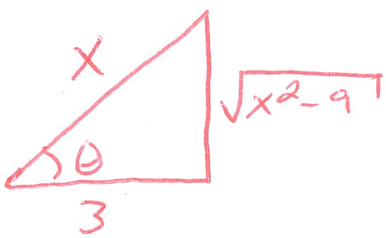
$$= \frac{1}{6}\int d\theta - \frac{1}{6}\int \cos(2\theta)\,d\theta$$

$$= \frac{1}{6}\theta - \frac{1}{12}\sin(2\theta) + C$$

$$= \frac{1}{6}\theta - \frac{1}{6}\sin\theta\cos\theta + C$$

$$= \frac{1}{6}\sec^{-1}\left(\frac{x}{3}\right) - \frac{1}{6}\cdot\frac{\sqrt{x^2-9}}{x}\cdot\frac{3}{x} + C$$

$$= \boxed{\frac{1}{6}\sec^{-1}\left(\frac{x}{3}\right) - \frac{\sqrt{x^2-9}}{2x^2} + C}$$



$x = 3\sec\theta$

$\frac{x}{3} = \sec\theta$

$\Rightarrow \sin\theta = \frac{\sqrt{x^2-9}}{x}$

$\cos\theta = \frac{3}{x}$

$\theta = \sec^{-1}\left(\frac{x}{3}\right)$

Please, show all work.

4. Compute the following indefinite integral.

$$\int \frac{x-9}{x^2+3x-10}\,dx$$

$$\frac{x-9}{x^2+3x-10} = \frac{x-9}{(x+5)(x-2)} = \frac{A}{x+5} + \frac{B}{x-2} \Rightarrow x-9 = A(x-2)+B(x+5)$$

Let $x=2 \Rightarrow 2-9 = A(2-2)+B(2+5)$
$-7 = 7B \Rightarrow B=-1$

Let $x=-5 \Rightarrow -14=-7A$
$\Rightarrow A = 2$

$$\Rightarrow \int \frac{x-9}{x^2+3x-10}\,dx = \int \frac{2}{x+5}\,dx + \int \frac{-1}{x-2}\,dx$$

$$= \boxed{2\ln|x+5| - \ln|x-2| + C}$$

5. Compute the following indefinite integral.

$$\int \frac{-x^2-x+9}{(x+2)(x^2+3)}\,dx$$

$$\frac{-x^2-x+9}{(x+2)(x^2+3)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+3} \Rightarrow -x^2-x+9 = A(x^2+3)+(Bx+C)(x+2)$$

$$\Rightarrow -x^2-x+9 = (A+B)x^2 + (2B+C)x + (3A+2C)$$

$$\Rightarrow \left.\begin{aligned} A+B &= -1 \\ 2B+C &= -1 \\ 3A+2C &= 9 \end{aligned}\right\}$$

3 eqns in 3 unknowns, use whichever method you like; substitution/elimination/matrix etc.

$$\Rightarrow A=1,\ B=-2,\ C=3$$

$$\Rightarrow \int \frac{-x^2-x+9}{(x+2)(x^2+3)}\,dx = \int \frac{1}{x+2}\,dx + \int \frac{-2x+3}{x^2+3}\,dx$$

$$= \int \frac{1}{x+2}\,dx - \int \frac{2x}{x^2+3}\,dx + 3\int \frac{1}{x^2+3}\,dx$$

$$= \boxed{\ln|x+2| - \ln|x^2+3| + \sqrt{3}\arctan\left(\frac{x}{\sqrt{3}}\right) + C}$$

Please, show all work.