LAST NAME: KEY FIRST NAME:

MATH 65B - Spring 2018

Groupwork 4: February 8, 2018

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1. Compute the following limits.

(a) $\lim_{x\to\infty} x^{\frac{1}{x}}$ $= \infty^{\frac{1}{\infty}} = \infty^0$ indeterminate

(b) $\lim_{x\to\infty} x\sin\left(\frac{\pi}{x}\right)$ $= \infty \sin\left(\frac{\pi}{\infty}\right) = \infty \cdot 0$ indeterminate

(c) $\lim_{x\to\infty} x^5 e^{-x^3}$ $= \infty \cdot 0$ indeterminate

(d) $\lim_{x\to 0^+} \frac{x^2}{x^5}$ $= \frac{\infty}{\infty}$ indeterminate.

a) $\lim_{x\to\infty} x^{1/x} = \lim_{x\to\infty} e^{\ln\left(x^{1/x}\right)} = \lim_{x\to\infty} e^{\ln(x)\cdot\frac{1}{x}}$

$\overset{\text{Thm.}}{=} e^{\lim_{x\to\infty} \frac{\ln(x)}{x}}$

$\overset{L'H}{=} e^{\lim_{x\to\infty} \frac{1/x}{1}} = e^0 = \boxed{1}$

b) $\lim_{x\to\infty} x\sin(\pi/x) = \lim_{x\to\infty} \frac{\sin(\pi/x)}{1/x}$ Let $t = \frac{1}{x}$, then as $x\to\infty$, $t\to 0^+$

$= \lim_{t\to 0^+} \frac{\sin(\pi t)}{t} \overset{L'H}{=} \lim_{t\to 0^+} \pi\cos(\pi t) = \boxed{\pi}$

c) $\lim_{x\to\infty} x^5 e^{-x^3} = \lim_{x\to\infty} \frac{x^5}{e^{x^3}} \overset{L'H(x5)}{=} \lim_{x\to\infty} \frac{5!}{9e^{x^3}x\left(27x^9+180x^6+240x^3+40\right)} = \boxed{0}$

(or argue why it must be zero)

d) $\lim_{x\to 0^+} \frac{x^2}{x^5} = \lim_{x\to 0^+} \frac{1}{x^3} = \boxed{\infty}$

Please, show all work.

2. Determine if the the integral is convergent or divergent. If it is convergent, compute the integral.

$$\int_0^\infty xe^{-5x}\,dx$$

$u = x \quad dv = e^{-5x}dx$

$du = dx \quad v = -\frac{1}{5}e^{-5x}$

$$\int_0^\infty xe^{-5x}dx = \lim_{t\to\infty}\int_0^t xe^{-5x}dx = \lim_{t\to\infty}\left(-\frac{1}{5}xe^{-5x}\Big|_0^t + \frac{1}{5}\int_0^t e^{-5x}dx\right)$$

$$= \lim_{t\to\infty}\left(-\frac{1}{5}xe^{-5x}\Big|_0^t - \frac{1}{25}e^{-5x}\Big|_0^t\right)$$

$$= \lim_{t\to\infty}\left(-\frac{1}{5}te^{-5t} - \frac{1}{25}e^{-5t} + \frac{1}{25}e^0\right)$$

$$= \boxed{\frac{1}{25}}$$

Convergent

3. Determine if the the integral is convergent or divergent. If it is convergent, compute the integral.

$$\int_{-\infty}^0 \frac{x}{1+x^2}\,dx$$

$$\int_{-\infty}^0 \frac{x}{1+x^2}dx = \lim_{t\to-\infty}\int_t^0 \frac{x}{1+x^2}dx$$

$u = 1+x^2$

$du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

$$= \lim_{t\to-\infty}\frac{1}{2}\int_{1+t^2}^1 \frac{1}{u}du$$

$$= \lim_{t\to-\infty}\frac{1}{2}\ln(u)\Big|_{1+t^2}^1 = \lim_{t\to-\infty}\frac{1}{2}\left(\ln(1) - \ln(1+t^2)\right)$$

$$= -\infty$$

divergent

Please, show all work.

4. Determine if the the integral is convergent or divergent.

$$\int_0^\infty \frac{\cos^2(x)}{x^2+1}\,dx$$

Use comparison test

Note that $|\cos^2(x)| \le 1$ and $\int \frac{1}{x^2+1}\,dx = \arctan(x) + C$

$$\int_0^\infty \frac{\cos^2(x)}{x^2+1}\,dx \le \int_0^\infty \frac{1}{x^2+1}\,dx$$

$$= \lim_{t\to\infty} \int_0^t \frac{1}{x^2+1}\,dx$$

$$= \lim_{t\to\infty} (\arctan(t) - \arctan(0))$$

$$= \lim_{t\to\infty} \arctan(t) = \frac{\pi}{2}$$

$$\Rightarrow 0 \le \int_0^\infty \frac{\cos^2(x)}{x^2+1}\,dx \le \frac{\pi}{2}$$

$\Rightarrow$ integral is convergent by Comparison Test.

5. Determine if the the integral is convergent or divergent. If it is convergent, compute the integral.

$$\int_0^2 x^2 \ln(x)\,dx$$

integrate by parts

$u = \ln(x)$ $\quad dv = x^2\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = \frac{x^3}{3}$

$$\int_0^2 x^2 \ln(x)\,dx = \lim_{t\to 0^+} \int_t^2 x^2 \ln(x)\,dx$$

$$= \lim_{t\to 0^+}\left(\frac{x^3}{3}\ln(x)\Big|_t^2 - \int_t^2 \frac{1}{3}\cdot x^2\,dx \right)$$

$$= \lim_{t\to 0^+}\left(\frac{8}{3}\ln(2) - \frac{t^3}{3}\ln(t) - \left(\frac{1}{9}x^3\Big|_t^2\right)\right)$$

$$= \lim_{t\to 0^+}\left(\frac{8}{3}\ln(2) - \frac{t^3}{3}\ln(t) - \frac{8}{9} + \frac{t^3}{9}\right)$$

$$= \boxed{\frac{8}{3}\ln(2) - \frac{8}{9} \quad \text{convergent}}$$

$$\lim_{t\to 0^+} \frac{t^3}{3}\ln(t) = \frac{1}{3}\lim_{t\to 0^+} \frac{\ln(t)}{1/t^3} \overset{L'H}{=} 0$$

Please, show all work.